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of

THE ROYAL AERONAUTICAL SOCIETY

OCTOBER 1958

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Journal of

THE ROYAL AERONAUTICAL SOCIETY

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All communications for publication in, and advertisements for, the JOURNAL *should be addressed to the Editor,* JOAN BRADBROOKE, *A.R.Ae.S.; all communications on general matters affecting the Society should be addressed to the Secretary,* A. M. BALLANTYNE, *T.D., B.Sc., Ph.D., Hon.F.C.A.I., A.F.I.A.S., F.R.Ae.S.*

None of the papers or paragraphs must be taken as expressing the opinion of the Council of the Royal Aeronautical Society unless otherwise stated.

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(Founded 1866)

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NOTICES

Seventh Anglo-American Conference

The tentative dates for the Seventh Anglo-American Aeronautical Conference are 4th-17th October 1959. Registration is expected to be on Sunday 4th October in New York and lectures will begin on 5th October. Following the lecture sessions in New York, visits will be made to centres in the United States and in Canada. The Conference is expected to end in Toronto, Canada on the 17th October 1959.

AGARD Meetings

The Eighth General Assembly of AGARD will open in Copenhagen, Denmark, on 28th October 1958, following Panel meetings, and will include a discussion on "The Impact of Space Technology on NATO Research and Development."

The meeting of the Structures and Materials Panel will be open to accredited observers from 20th to 23rd October. The major part of the programme will be on structures for high speed flight. Further details are available from A. J. Barrett, the Royal Aeronautical Society.

A meeting on "Engine Test Facilities and Techniques" will be held jointly by the Combustion and Propulsion, Wind Tunnel, and Flight Test Panels on 23rd and 24th October. The programme will include consideration of propulsion systems test requirements, means of obtaining test conditions for altitude simulation, instrumentation and measurement of steady state and transient conditions, scaling, component testing, engine flight testing, and correlation of results obtained by all methods.

New Digital Computer Techniques

Specialist Discussion Meetings on New Digital Computer Techniques are to be held at the Institution of Electrical Engineers on 16th and 17th February 1959. The meetings will cover the electrical engineering aspects of digital computers and provisionally the Sessions will deal with the following subjects:— *Session* 1. Character Recognition. *Session* 2. Peripheral Equipment (magnetic tape, paper tape, fast printers, and other forms of input and output devices; and problems arising in their attachment to digital computers). *Session* 3. Low Temperature Storage and Switching Devices. *Session* 4. Special Aspects of Logical Design, for example, logical circuit techniques, fast carry logic for adders, interrupt facilities, and the relation of logical design to reliability.

The meetings will be open to members and non-members of the Institution. Offers of contributions should be sent to, and particulars about the meetings may be obtained from, The Secretary, The Institution of Electrical Engineers, Savoy Place, London W.C.2.

Conference on Interfacial Phenomena

The Winter Conference of the Physical Society on physics of the solid state, which will be on Interfacial Phenomena, will be held at the Cavendish Laboratory, Cambridge on 18th, 19th and 20th December 1958. Accommodation will be available in King's College at a charge of 35s. per day (including meals).

It is hoped that papers will be given on some or all of the following subjects:—

1. Structure of vacuum evaporated layers on single crystal substrates.
2. Structure of electro-deposited layers.
3. Epitaxy; orientation of crystals on various substrates.
4. Grain boundary phenomena; the interface between different phases in metals.
5. The interface between solid and liquid phases.

Investigators who would like to contribute papers should write to Professor N. F. Mott, Cavendish Laboratory, Cambridge. Further particulars of the Conference may be obtained from the Secretary, The Physical Society, 1 Lowther Gardens, Prince Consort Road, London S.W.7.

Lectures—Changes of Venue and Date

Since the Lecture Card for the 1958-9 session was printed changes have had to be made in the arrangements for several of the Lectures.

Dr. J. Williams' Lecture on "Boundary Layer Control to Produce High Lift at Low Speeds" on 11th November 1958 and the lecture by Mr. S. S. Hall and Mr. A. Thomas on "Application of a Constant Frequency A.C. Electrical System to Aircraft and Missiles" on 9th December will now be held in the Theatre, PARK LANE HOUSE, 45 PARK LANE, W.1. (just past Curzon Street, entrance around the corner in Stanhope Gate, near the Dorchester).

The Joint Lecture with the Helicopter Association on 7th November 1958 "Development of the Fairey Rotodyne" by Dr. G. S. Hislop will now be held **at the Institution of Civil Engineers,** Great George Street, London S.W.1 at 7 p.m.

The Guided Flight Section Lecture "Drone Aircraft" by H. G. Conway which was to have been given on Monday 15th December 1958 will now be given on **WEDNESDAY 17th DECEMBER 1958.** Time and venue unchanged.

Henlow Cadet Prize

The Henlow Cadet Prize for 1958 has been awarded to Pilot Officer A. J. Lowery. The Prize is three years' subscription as a Graduate of the Society, with entrance fee, and a book.

Associate Fellowship Examination—December 1958

The next Associate Fellowship Examination will be held on Tuesday, Wednesday and Thursday the 16th, 17th and 18th December 1958. All candidates will be sent a detailed time-table.

Acknowledgment

The Council wishes to thank C. G. F. Brown, *Associate Fellow,* for presenting to the Library Manuals of Rolls-Royce "Eagle" and "Falcon," Wolseley "Viper" and "Liberty" Aero Engines.

Associate Fellowship Examination Results

The following Candidates were successful in the Associate Fellowship Examination held in June 1958.

Part I—United Kingdom

R. F. Barton. Pure Mathematics, Mechanics, Physics, Strength of Aircraft Materials.

A. S. Chakrapany. Strength of Aircraft Materials, Theory of Machines (Distinction); N. J. Chamberlain. Strength of Aircraft Materials (Distinction); R. Cloke. Aerodynamics.

A. W. Delobelle. Pure Mathematics, Mechanics, Physics, Theory of Machines.

I. J. Ford. Pure Mathematics, Mechanics, Physics, Aerodynamics, Strength of Aircraft Materials.

C. M. Jagtiani. Strength of Aircraft Materials.

J. Kime. Aerodynamics.

A. C. Langmead. Aerodynamics, Thermodynamics; D. J. Loveridge. Aerodynamics.

Z. Mahmood. Pure Mathematics, Mechanics, Thermodynamics, Theory of Machines; J. R. Morgan. Aerodynamics.

J. F. North. Aerodynamics.

J. N. Puckering. Aerodynamics.

I. W. Strachan. Aerodynamics, Thermodynamics; C. J. Sturt. Aerodynamics, Thermodynamics.

P. M. Toor. Theory of Machines.

A. A. G. Woodford. Aerodynamics (Distinction), Thermodynamics.

PART I—ABROAD

P. KUNHIRAMAN (Poona). Thermodynamics.

S. K. MAJUMDAR (Calcutta). Pure Mathematics, Physics, Strength of Aircraft Materials, Thermodynamics.

B. S. B. NAIDU (Mysore). Thermodynamics.

K. G. K. PILLAI (Delhi). Mechanics, Physics, Thermodynamics, Theory of Machines.

P. M. RAMACHANDRAN (Calcutta). Physics, Aerodynamics, Strength of Aircraft Materials (Distinction).

V. V. SAMUEL (Madras). Pure Mathematics, Physics, Thermodynamics, Theory of Machines; J. SEN (Calcutta). Thermodynamics; B. T. SWEENEY (Point Cook, Australia). Aerodynamics (Distinction), Thermodynamics.

K. THARASANKAR (Delhi). Thermodynamics, Theory of Machines (Distinction).

J. G. WILSON (Vancouver Island). Thermodynamics.

PART II

B. HUSAIN (Karachi). Thermodynamics and Theory of Machines.

PART II—UNITED KINGDOM

N. W. BEATTIE. Theory of Structures A, Theory of Structures B, Theory of Structures C; A. A. BUTLER. Theory of Structures A, Theory of Structures B, Theory of Structures C.

A. H. COMBER. Theory of Structures B.

J. ELLIOTT. Theory of Structures B.

A. G. FOSKETT. Theory of Structures A, Theory of Structures B (Distinction), Theory of Structures C (Distinction).

L. F. GILLARD. Theory of Structures C; A. R. GUY. Theory of Structures A, Theory of Structures B.

S. IRVING. Theory of Structures A, Aircraft Design and Development A.

G. MCCURDY. Theory of Structures A (Distinction), Theory of Structures B, Theory of Structures C.

P. G. RAYMONT. Aerodynamics A, Aerodynamics B.

J. STRYMOWICZ. Theory of Structures A, Theory of Structures B.

J. P. H. WEBBER. Theory of Structures A, Theory of Structures B, Theory of Structures C; G. R. WEBSTER. General Design Piston and Turbine Engines, Piston and Turbine Engines.

DIARY

LONDON

9th October

THE FOURTEENTH BRITISH COMMONWEALTH LECTURE.—The Canadian Approach to All-Weather Interceptor Development. J. C. Floyd. Royal Institution, 21 Albemarle Street. 6 p.m. (Tea at 5.30 p.m.)

27th October

LECTURE*—The Bloodhound. D. J. Farrar. Institution of Civil Engineers, Great George Street, S.W.1. 6 p.m. (Tea at 5.30 p.m.)

4th November

LECTURE—The Spectre Rocket Engine. W. N. Neat. Library, 4 Hamilton Place. 7 p.m.

7th November

JOINT LECTURE WITH THE HELICOPTER ASSOCIATION—Development of the Fairey Rotodyne. Dr. G. S. Hislop. **Institution of Civil Engineers, Great George Street, S.W.1. NOTE CHANGE OF VENUE.**

11th November

LECTURE—Boundary Layer Control to Produce High Lift at Low Speeds. Dr. J. Williams. **The Theatre, Park Lane House, 45 Park Lane, W.1. 7 p.m. NOTE CHANGE OF VENUE.**

***Arranged by the Committee of the Guided Flight Section.**

20th November

THE SECOND LANCHESTER MEMORIAL LECTURE.—Aeroelasticity—Retrospect and Prospect. Professor A. R. Collar. Institution of Mechanical Engineers, Birdcage Walk, S.W.1. 6 p.m. (Tea 5.30 p.m.)

GRADUATES' AND STUDENTS' SECTION

16th October

The Fatigue of Aircraft. Major P. L. Teed. Library, 4 Hamilton Place. 7.30 p.m.

29th October

Annual Film Show. Library, 4 Hamilton Place. 7.30 p.m.

11th November

The Air Transport Auxiliary. Sir Gerard D'Erlanger. Library, 4 Hamilton Place. 7.30 p.m.

28th November

Annual Dance. Library, 4 Hamilton Place. 7.30 p.m.

BRANCHES

13th October

Glasgow.—Graduates' and Students' Section Opening Meeting—Chairman's Address and Film Show. College of Science and Technology, Duke Street. 7.30 p.m.

15th October

Bristol.—Presidential Address. Filton House. 6 p.m.

Coventry.—Digital Computers. R. F. King. The Wine Lodge, The Burges. 7.30 p.m.

Isle of Wight.—What Forces can the Human Body Withstand? Flt. Lt. J. C. Guignard. Clubhouse, Saunders-Roe Sports and Social Club, Church Path, E. Cowes. 6 p.m.

20th October

Glasgow.—Propulsion Systems for Helicopters. Dr. A. W. Morley. College of Science and Technology, Duke Street. 7.15 p.m.

Henlow.—Guidance and Control. L. H. Bedford. Building 62, R.A.F. Technical College. 7.30 p.m.

21st October

Luton.—Fatigue Testing of the Fokker "Friendship." E. J. van Beek. Napier Senior Staff Canteen, Luton Airport. 6.15 p.m.

23rd October

Bristol.—Film Show. Filton House. 6 p.m.

Southend.—Air Safety—The Work of the Air Registration Board. W. Tye. Labour Hall, Boston Avenue. 7.30 p.m.

29th October

Christchurch.—50 Years of Aviation. G. Errington. King's Arms Hotel. 7.30 p.m.

Hatfield.—Some Recent Developments in Aircraft Casting Manufacture. R. W. Eade. de Havilland Restaurant. 6.15 p.m.

Gloucester and Cheltenham.—Astronomical Research and Space Travel. A. F. Collins. Meeting to be in Hereford. 7.30 p.m.

London Airport.—Brains Trust. Senior Mess Club, B.O.A.C. Headquarters, L.A.P. 6 p.m.

Weybridge.—Brains Trust. Apprentice Training School, Vickers-Armstrongs (Aircraft) Ltd. 6.10 p.m.

30th October

Belfast.—Some Applications of High Nickel Alloys in the Aircraft Industry. W. H. Featherstone. Grand Central Hotel. 7 p.m.

3rd November

Cheltenham.—Instruments for Guided Flight. G. G. Roberts. Rotunda, Cheltenham. 7.30 p.m.

Derby.—The Third Sir Henry Royce Memorial Lecture—The Present and the Future. Air Cmdre. F. R. Banks. Rolls-Royce Welfare Hall, Nightingale Road. 6.15 p.m.

Henlow.—Operation Grapple. Group Capt. R. H. C. Brousson. Building 62, R.A.F. Technical College. 7.30 p.m.

4th November

Luton.—Guided Flight. L. H. Bedford. Napier Senior Staff Canteen, Luton Airport. 6.15 p.m.

6th November

Bristol. — Junior Committee Lecture — Helicopters. R. Hafner. Filton House. 6 p.m.

Isle of Wight.—Annual Dinner, Hotel Ryde Castle.

11th November

Boscombe Down.—The Application of an A.C. Electrical System to Aircraft. S. S. Hall. A. & A.E.E. Lecture Hall. 5.45 p.m.

12th November

Bristol.—Joint Meeting with the I.P.E.—Some Thoughts on the American and British approach to Aircraft and Aero-Engine Procurement and Production. Air Cmdre. F. R. Banks. Filton House. 6 p.m.

Hatfield.—Aviation, Medicine and Air Crew Personal Equipment. Ft. Lt. J. King. de Havilland Restaurant. 6.15 p.m.

Manchester.—Digital Differential Analysers. K. V. Diprose. Reynolds Hall, College of Technology. 7.30 p.m.

Reading.—Film Show. Canteen, Western Manufacturing Ltd. 6.15 p.m.

Southend.—Film Show. Labour Hall, Boston Avenue. 7.30 p.m.

13th November

Swindon.—The Fairey Rotodyne. D. M. Davies. The College, Victoria Road. 7 p.m.

17th November

Halton.—Flight Refuelling. P. G. Proctor. Branch Headquarters, R.A.F. Halton. 6.45 p.m.

Henlow.—The Objects and Methods of Aerial Survey. J. H. Saffery. Building 62, R.A.F. Technical College. 7.30 p.m.

18th November

Bristol.—Ram-jets. Dr. R. R. Jamison. Filton House. 6 p.m.

Luton. Discussion Paper Evening. Napier Senior Staff Canteen, Luton Airport. 6.15 p.m.

News of Members

A. J. Alexander (*Graduate*) has taken up a Research Fellowship at the College of Aeronautics and will study problems related to Boundary Layer Control and Jet Flaps.

Squadron Leader J. A. Beadle (*Associate Fellow*) formerly with the Directorate of Air Intelligence, Ottawa has been posted to the R.A.F. Station, Scampton, Lincs.

H. C. Bellamy (*Associate Fellow*) Inspector-in-Charge, A.I.D. (M.O.S.) Lockheed Hydraulic Brake Company, has retired owing to ill-health.

Maxwell K. Bull (*Graduate*) has left the de Havilland Aircraft Co. Ltd., and taken up a Research Fellowship in the Department of Aeronautical Engineering, Southampton University.

J. T. Burr (*Associate Fellow*) has taken up a new appointment with Bristol Aircraft Ltd., as Systems Engineer —Projects.

D. Chapman (*Graduate*) is now working as an Engineer in the Atomic Power Division of the English Electric Company at Whetstone, near Leicester.

A. V. Cleaver (*Fellow*) formerly Assistant Chief Engineer, Rocket Propulsion, at Rolls-Royce (Aero Division) has been appointed Chief Engineer.

Gerald Clerehugh (*Graduate*) has joined the staff of the Blackburn and General Aircraft Co. Ltd., at Brough.

D. A. Collings (*Student*) has been appointed a Stress Technician with A. V. Roe and Co. Ltd., in the Weapons Research Division at Woodford, Cheshire.

A. L. Cresswell (*Associate*) formerly Senior Design Draughtsman in the Aircraft Division of Short Brothers and Harland Ltd., has been appointed Section Leader (Reliability) in the Guided Missiles Division of the firm.

John J. Cripps (*Associate Fellow*) has been appointed Senior Lecturer in the Mechanical Engineering Department, Brunel College of Technology, Middlesex. He has also been awarded the Diploma of Membership of the Imperial College.

R. E. Daisley (*Graduate*) has left Bristol Aero Engines Ltd. and is now employed as a Research Physicist by Shell Research Ltd. in their Technical Information Section at Thornton Research Centre, Cheshire.

S. D. Davies (*Fellow*) formerly Managing Director of Dowty Fuel Systems has rejoined the Hawker Siddeley Group to take charge of engineering work on a new project.

G. E. Dodd (*Associate Fellow*) formerly of Air Technical Publications, M.O.S., has been appointed Senior Reliability Engineer with the Guided Missile Division of the de Havilland Aircraft Company of Canada, Toronto.

Wing Commander B. E. Dunkley (*Associate Fellow*) has joined the staff of the English Electric Co. Ltd., Guided Weapons Division, as an Engineer.

Mrs. D. H. Fairweather (*Graduate*) formerly part-time assistant science mistress at Mellow Lane Comprehensive School, Hayes End has been appointed part-time Applied Mathematics Mistress at the High School for Girls, Slough.

J. S. Fowler (*Associate*) formerly of Bristol Aircraft Ltd., has been appointed an Engineer Instructor with British European Airways.

I. V. Franklin (*Associate Fellow*) formerly Assistant Chief Stressman at British Messier Ltd., is now Head of the Technical Section, Mechanical Engineering and Structures, English Electric Company, Guided Weapons Division, London Design Office.

R. C. Godwin (*Associate Fellow*) formerly Deputy Chief Designer of Marshalls Flying School has joined de Havilland Propellers Ltd., as Senior Engineer. (Correction from September Journal Notices.)

L. S. Greenland (*Fellow*) Chief Designer and Director of H.M. Hobson Ltd., has been appointed a Director of Integral Ltd.

Lieut. Commander R. P. Grimwood (*Associate Fellow*) has retired from the Royal Navy and joined British Hydrocarbon Chemicals Ltd.

Alan J. Harland (*Graduate*) formerly a Stressman at the Dumbarton Office of Blackburn and General Aircraft Ltd., is now employed as a Stressman with H.M. Hobson Ltd., Wolverhampton.

F. A. Harrington (*Associate*) has resigned his position as a Technical Representative with the de Havilland Aircraft Co. Ltd., and joined the Ford Motor Company as a Development Engineer.

A. H. Higgs (*Associate Fellow*) formerly with the Bristol Aeroplane Co. Ltd., is now on the staff of the Lakehead College of Arts, Science and Technology at Port Arthur, Ontario.

F. H. Hooke (*Associate Fellow*) having completed two years' research in fatigue of metals at the University of Nottingham is returning to the Aeronautical Research Laboratories, Melbourne to take charge of investigations into the life of aircraft structures.

James Carson Howard (*Associate Fellow*) formerly of the Structures Department of the Lockheed Aircraft Corporation, Burbank, California is now Structures Engineer in the Missiles Systems Division of the same company.

Squadron Leader W. E. B. Hurst (*Associate*) has left the R.A.F. Technical College and taken an appointment in the Guided Weapon Squadron of the Central Servicing Development Establishment at R.A.F. Swanton Morley, Norfolk.

Air Commodore L. E. Jarman (*Associate Fellow*) has left the Royal Air Force and is now Secretary-General to the Engineering Industries Association.

S. H. Jones (*Associate Fellow*) has left his former post of De-Icing Administration Engineer with D. Napier and Son Ltd., Luton, and joined Henry W. Peabody (Industrial) Ltd., as Deputy to the Director in charge of the Ransburg Division.

W. A. Lemmer (*Graduate*) has left Bristol Aircraft Ltd., and is now in the Stress Office of the English Electric Company at Warton.

Eric Litton (*Graduate*) formerly a Stress Engineer with de Havilland Aircraft Co. Ltd., has been appointed a

Lecturer in Mechanical Engineering at Kingston Technical College, Surrey.

C. G. LONGFORD (*Associate Fellow*) formerly Chief Flight Test Engineer, Gloster Aircraft Co. Ltd., is now Technical Director, Wynn Developments Ltd., Staverton Aerodrome.

P. G. LUCAS (*Fellow*) has been appointed Assistant to the Managing Director of the de Havilland Aircraft Co. Ltd.

A. R. METTAM (*Associate Fellow*) has been transferred from the R.A.E., Bedford on his secondment to the Ministry of Health to carry out Engineering Research on artificial limbs at Roehampton Hospital.

R. W. MCKAY (*Associate Fellow*) formerly Deputy Inspector in Charge A.I.D., de Havilland Aircraft Co. Ltd., has been transferred to the Aircraft Division M.O.S./A.I.D., South Eastern Region.

J. MCNAUGHTON (*Graduate*) formerly a Development Engineer with de Havilland Aircraft of Canada Ltd., is now a Project Engineer with the Garrett Manufacturing Corporation of Canada Ltd.

J. C. MCVEIGH (*Graduate*) formerly Lecturer in Mechanical Engineering, Kumasi College of Technology, Ghana is now a Lecturer in Mechanical Engineering at the Borough Polytechnic, London.

A. R. B. NASH (*Associate Fellow*) has resigned his position as Research Engineer at the Research and Development Department, Vickers-Armstrongs (Aircraft) Ltd., Weybridge and has become a Director of Kinetrol Ltd., Control Engineers.

B. G. NEWMAN (*Associate Fellow*) formerly at the Engineering Laboratory, Cambridge University has been appointed to the Canadair Research Professorship in Fluid Mechanics at Laval University, Quebec for 1958-59.

Major G. L. NEWTON-WADE (*Associate*) has been appointed Senior Technical Officer to the Army Corps in Middle East Land Forces. This appointment is in addition to his appointment as Officer Commanding, Light Aircraft Workshop, R.E.M.E., 653 Squadron, Army Air Corps.

Squadron Leader F. J. O'CONNELL (*Associate*) has retired from the Royal Air Force and taken up a post with the G.Q. Parachute Company, Woking.

E. I. RACZNSKI (*Associate Fellow*) formerly Chief Experimental Engineer of Industrias Argentina has been appointed Chief Engine Engineer of the same firm.

DENIS F. REARDON (*Student*) has left Vickers-Armstrongs (Aircraft) Ltd., at Weybridge on his award of a College Research Assistantship at Battersea College of Technology.

F. A. ROBERTS (*Associate Fellow*) formerly with British Overseas Airways Corporation is now Contracts Manager, Sperry Gyroscope Co. Ltd., Bracknell Division.

Wing Commander A. J. H. SAMMELS (*Associate*) has joined the Sperry Gyroscope Co. Ltd., at Bracknell, Berks.

D. M. SQUIRES (*Graduate*) has returned from the California Institute of Technology and has been appointed a Senior Engineer (Propulsion) at the London Design Office of the de Havilland Propeller Co. Ltd.

K. G. STAPLEY (*Graduate*) has obtained the Diploma of the College of Aeronautics, Cranfield and is now Technical Assistant in the Flight Test Development Section of Blackburn and General Aircraft Ltd., Brough.

PETER A. E. STEWART (*Associate*) formerly a Design Engineer at the Rocket Division of Bristol Aircraft is now a Development Engineer in the Rocket Development Section of Saunders-Roe Ltd.

R. H. TRELEASE (*Associate*) formerly at the Georgia Division of the Lockheed Aircraft Corporation has been appointed Research Engineer with the Missile Systems Division of Lockheeds at Palo Alto, California.

Lieut. Commander C. F. WHITTAKER (*Associate*) has been appointed Technical Instructor in the Instruction School at Rotol and British Messier Ltd., Gloucester.

Flt. Lieut. E. S. F. WRIGHT (*Associate Fellow*) formerly of the R.A.F., is now a Student at the College of Aeronautics, Cranfield for the two-year Diploma Course.

INTERNATIONAL CONFERENCE ON INFORMATION PROCESSING

An International Conference on Information Processing, organised by UNESCO with the help of a group of consultants from various countries, is to be held in Paris from 15th to 20th June 1959. The programme of the Conference will comprise Six main headings:— 1. Methods of digital computing (including numerical analysis). 2. Logical design of digital computers. 3. Common symbolic language for digital computers. 4. Automatic translation of languages. 5. Collection, storage and retrieval of information. 6. Patterns recognition and machine learning.

An international exhibition of data processing equipment will be held in Paris at the time of the Conference and visits will be organised to factories manufacturing such equipment in the Paris area and to computation laboratories in Paris and possibly in neighbouring countries.

In the United Kingdom enquiries about the Conferene should be made to: The Honorary Secretary, Group B-Computation and Automatic Control, British Conference on Automation and Computation, c/o The Institution of Electrical Engineers, Savoy Place, London W.C.2.

ELECTIONS

The following is a list of new members and transfers of membership of the Society.

Associate Fellows

Frank Russell Beach (*from Graduate*)
Bernard Albert Chacksfield
John Charles Cooper (*from Graduate*)
Raymond Cuthbert Davison
Brian Eales
Brian Keith Glenister (*from Graduate*)
Philip Ian Ivelaw
Jakob Daniel Theron Louw
John Alastair Douglas McEwen
Donald McKay McWilliam
Wilfred Edwin Pearce (*from Graduate*)
Arthur Jack Smaill (*from Associate*)
Ann Courtenay Welch (*from Associate*)

Associates

Henry Stiles Allen
Percy Frank Barratt
John Goodwin Burns
Martin Frederick Richard Burton
Colin Jasper Gardiner
Stuart Ross Hawkey
George Alexander Naylor
Frederick Ernest Frank Prince
Ronald Albert Springall

Graduates

Kenneth Archibald Campbell
Michael William Sayle Grigson
Janet Mary Gulland
Peter Brian Lockwood
Tirumalai Lakshmi Narasimhan
Hugo Rowton Simpson (*from Student*)

Students

George Bernard Avison
John Roderique Marriott Bull
Alain W. Delobelle
Geoffrey Laurence Alfred Dray
Michael John Hide
Frank Joseph Kinsella
Brian Wilfred Lowrie
Adrain Millward
Robert William Haviland Minchin
Robin Anthony Newnham
Michael Schofield
James Smith Somerville
Colin John Hudson Veale
Malcolm John Watson

ERRATUM—THE MISSING X'S

In numbering the pages of Notices in the July JOURNAL unfortunately an X was dropped and the numbering of the Notices pages in the July, August and September JOURNALS duplicates those of May and June. The Missing X has now been found and replaced in this JOURNAL.

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EDWARD PEARSON WARNER

M.Sc., B.A., D.Sc.(Hon.), LL.D.(Hon.), Hon.F.I.A.S., Hon.F.C.A.I., *Honorary Fellow*

1894 - 1958

An Appreciation

by

T. P. WRIGHT, Hon.F.I.A.S., *Honorary Fellow*
(*Vice-President for Research, Cornell University*)

THERE HAVE BEEN few men who, over so long a period of time, have contributed so much in breadth and depth to their chosen fields as did Ed Warner to aviation. In time, the extent was about 45 years. In breadth, his range of activities included all scientific and technological areas that impinge on aeroplane design; air transport operations including air navigation and economics; aeronautical writing and editorship; public service in many fields; and international administration and statesmanship. In depth, his abilities penetrated to fundamentals in all activities he undertook. Thus, his accomplishments were truly three-dimensional; and with all, he was a man of great sympathy, tolerance and understanding of the views of others, and one whose great powers of speech usually induced others to accept his point of view. Dr. Hugh Dryden, Director of the U.S. National Advisory Committee for Aeronautics, Hon.F.R.Ae.S., and Wilbur Wright Lecturer, has the following to say:

> "In the passing of Edward Pearson Warner, the aeronautical world has lost one of its greatest men. Ed Warner's capacity and passion for knowledge, his prodigious energy, and the wide range of his interests resulted in major contributions to all aspects of aeronautics, technical and non-technical, national and international. Among his friends he was known as America's aeronautical encyclopaedia, for his memory of things aeronautical was phenomenal. He was able to speak fluently and logically at short notice on a multitude of topics, but at a speed which distressed the reporters who tried to record his remarks. His influence on the development of aeronautics in all its aspects was profound."

In dealing with this great man, it appears desirable to consider separately several phases of his career as they developed and overlapped and I have here adopted that plan. Although I was myself well aware of most of his activities through a period of 40 years and quite closely associated with him in some, it is obvious that others were closer to him in many. Consequently, I have written to a number of his distinguished friends seeking information from their recollections of his work in those areas in which they were associated with him.

Early Years

Edward Pearson Warner was born on 9th November 1894 in Pittsburgh, Pennsylvania. At an early age, his family moved to Massachusetts, first residing in Concord and then Cambridge. One can, I believe, best acquire a feeling for his background by considering the remarks of a boyhood friend, Raymond P. Baldwin, who has kindly supplied the following:

> "When I first met him, we were in the fourth class at Volkmann School in Boston, and within six weeks of being fourteen years old. He had a cowlick, an unmilitary slouch, and the warmest and most friendly big smile I can remember. He was somewhat clumsy, physically, and wore custom-made brown suits that looked as though they had been slept in. He enjoyed other boys' pranks, but did not indulge in any of his own. His marks were all good; but in mathematics, he was phenomenal. He did algebra problems in his head.
>
> "Before we took our elementary algebra examinations for Harvard, Mr. Volkmann went to Cambridge to see the examiners and explain Ed, so that they wouldn't think he had cribbed if he put down almost nothing but the final answers. We were allowed an hour and a half for the examination which was held at the school. Ed finished it in twenty minutes, handed in his blue book, and then went down to the boss's office, where Mr. Volkmann and one of the instructors were working out the answers. Ed waited for them to finish, and then compared his answers with theirs. One of his differed, and he said, 'Oh, shucks; in the seventh problem, I forgot to multiply by Pi!'
>
> "Ed's high forehead and well-shaped head indicated the raw material for an intellect; but his early skill with mathematics he attributed to a grandmother to whom he had gone at the age of six to get help with his arithmetic. She struggled with

the problem for a while, and then said, 'Edward this is very hard to do in arithmetic; let me show you how much easier it is in algebra.'

"We didn't see much of him after school hours. I don't recall his having gone out for athletics, except the shot-put, and some rather clumsy tennis with friends. He was strong, and invaluable as anchor man in a tug-of-war.

"He lived in Concord, nearly twenty miles away and commuted by motorcycle. The Warner place, on Elm Street in Concord, was an estate of perhaps eighty acres. The large brick house, some five hundred feet back from the street with a tennis court in the foreground, had two large living rooms separated by a broad hallway. In the rear, the land stretched through the woods to the Sudbury River.

"About this time, Ed's parents moved to Cambridge, and I spent part of one summer with Ed and his father on Brattle Street. Robert Warner in his late forties looked like a man in his early thirties. He was shorter than Ed, but erect, dark haired, self-confident and positive. He was an engineer, and was of the opinion that engineers were the only people who knew how to run things.

"It would be only a guess that when vacationing on the Cape, Ed spent part of his time watching the soaring flight of the sea gulls; but it is no guess that he was always interested deeply in flight—soaring and propelled. While still in school—in 1911, I think—he and another classmate built a glider and entered it in a meet in Boston. They won with a record flight of over 1,000 ft. I didn't see the meet, but would guess that this friend was the boy in the air; for it is my impression that Ed Warner never had the co-ordination necessary to permit him to get a pilot's licence. He wasn't touchy about his clumsiness; and would lead the laughter it sometimes caused at his own expense. But I'm inclined to think that was due more to character than to lack of sensitivity and chagrin.

"He went into college with honors, and roomed with another friend at Westmorley Hall. In college, at least in his own quarters, he studied with his old brown felt hat on.

"In 1921, because I was a lawyer, he came to me with the problem of getting a statute passed by the Massachusetts legislature to govern the licensing of pilots and the registration and regulation of aircraft. Together we wrote the bill which in 1922 became law, as an addition to Chapter 90 of the General Laws.

"Late in 1925, Ed asked me to go with him to East Boston. The Mayor had indicated an intention to sell the filled land for industrial building sights. Ed foresaw the great advantage to Boston in having an airport so close to the heart of the city. As a result of our visit, we formed the Boston Airport Corporation to take over and extend existing operations, and the Airport Development Company to build a supposedly permanent hangar. The legislature appropriated $35,000 for cinders to spread on the runways, though the Chairman of Joint Ways and Means said we were crazy to think it would ever amount to anything. Boston's best responded nobly to an appeal for funds. Our list of stockholders read like a composite of Dun and Bradstreet's highest ratings and the Social Register. Fortunately, when the big airlines moved in, all our investors were paid off with a handsome profit.

"Ed Warner relied upon his intellect. He felt safest with figures; and even with figures, he was not given to speculation. To him, two and two made four—never twenty-two. I once suggested to him that every cell in the human body had a modicum of brain. No—to him the brain was all in the head. In the early 20's, I spoke of the possibility of trans-Atlantic flight. Ed proved, with supporting figures, that the inherent limitations of the internal combustion motor made it impossible to carry enough gas to make it.

"He may have changed his mind about this later; for at an Archie Club dinner, he told me that Pratt and Whitney had come out with a new radial engine that weighed only $1\frac{1}{2}$ lb. per horse power as against $2\frac{1}{2}$ for the Wright Whirlwind. (The Niles-Bement and Pond stock that I bought in consequence jumped about thirty times in value within the next few months.)

"Sometime along in here, Ed spent a weekend with his friends, the Dwights. During the night there was a thunderstorm. Lightning struck a tree in front of their house and set it afire. Ed took a kettle full of water and climbed above the fire to quench it. Unfortunately, he poured the water on an exposed power line. The resulting shock knocked him out of the tree and broke his ankle, which had to be made immobile. He told me later that while falling, he estimated the voltage as between four and six thousand volts. (Note: This accident caused the limp which persisted through his lifetime.)

"Ed's subsequent career is set forth in *Who's Who*. Sheer merit led him from one thing to another, until the final triumph of his Presidency of I.C.A.O. His close British friend, Philip Teed, told me that Ed Warner knew more about aviation than anyone else, not just engineering but administration and law and all the problems involved in the job."

Several of the qualities later so evident in his make-up—integrity, ability to concentrate, mathematical genius, unimportance of clothes, clumsiness, early interest in aviation, appear in this description of his early years. A few other points of significance have been gleaned from other sources. For example, in 1916, he published technical articles on aviation while an undergraduate at Harvard.

Ed saw his first aeroplane in September 1910 and it seems to have been love at first sight, motivating him to develop the prize-winning glider.

Warner was graduated from Harvard with honours in 1916 with a B.A. degree, continuing his work there and at the Massachusetts Institute of Technology, then referred to as Boston Tech, to get a B.S. in 1917. In 1919, he received his Master's degree from the latter

Karsh, Ottawa

EDWARD P. WARNER

institution. Some nineteen years later (1938), he was awarded an honorary Doctor of Science degree by Norwich University.

Teacher

Edward Warner began his teaching activities as an instructor while still attending M.I.T. as a student and candidate for a Master's degree. It was during this period, the winter of 1917, when, as a cadet in the U.S. Naval Reserve Flying Corps detachment, I first came in contact with him. He gave the course on propellers. It consisted of several lectures during which he introduced us to the theories of Drzewiecki and Froude and the reasoning necessary to combine the blade element theory of the one and the inflow-outflow concepts of the other. I'm afraid we didn't absorb too much from his rapid-fire lectures, but we were tremendously impressed by his knowledge and fluency. Among the many who were his students in these courses for Army and Navy aviation cadets were men who later made their mark in aviation in the United States such as Captain Walter Diehl, Leroy Grumman, Charles Chatfield, Charles Montieth and Captain Ralph Barnaby.

During 1919 and 1920, Dr. Warner's teaching activities were interrupted by scientific work for N.A.C.A., both in the U.S. at Langley Field, Virginia, and abroad. However, he returned to M.I.T. as an Associate Professor of Aeronautical Engineering in 1920 and continued his teaching career there until 1926. Many outstanding figures in American aviation were his pupils, including Generals James Doolittle, Leslie MacDill and Frank Carroll; Admirals Felix Stump, M. F. Schoeffel and Theodore Lonnquist; George Chapline, Mac Short, Otto Kirchner, Dan Sayre, John G. Lee, Otto Koppen, James McDonnell, Arthur Raymond, William Stieglitz, Korvin-Kroukovski and Paul Johnston.

While at M.I.T., Miss Elizabeth Brown, now Historical Librarian of the I.A.S., was Ed's secretary (as she was later in Washington). She recalls, "In the 1920's, the aeronautical course at M.I.T. was a graduate one, and E.P.'s lectures were so difficult that the boys said that they should have chipped in and hired a stenographer to take notes for them. As we all know, he was a mathematical genius." Paul Johnston, Director of the Institute of the Aeronautical Sciences, wrote to me:

> "I worked under him in my senior year, 1920-21, at M.I.T. He was just then developing the course and the only thing that was available for me to take was an aeronautics option in my senior year which I did. I also did my thesis for Ed—the subject being 'Thin Wall Steel Tubes as Long Columns.' You will recall that during the post-war period, welded steel tube fuselage was just coming in and Ed was interested in having as much design data as he could get on such things.
>
> "During this period, he certainly was the aloof young professor and I had very few contacts with him except about thesis matters. I must say that in spite of the fact that the age difference between us was only about five or six years, it was rather a formidable job to go in to see him on any subject. This again, was a by-product of feeling that he knew a lot more about everything than all of the rest of us put together. He had a remarkable capability of quick reading and almost total recall of the things that he had read. He had as close to an encyclopaedic a mind as anyone I have ever known."

Again Miss Brown says, "We used to call him a 'dynamo' in trousers at Tech. He always went around with his pockets bulging with papers. He had special pockets made in his suits for this purpose." And Paul Johnston recalls, "He usually had an armful of papers and magazines wherever he went and he always had a line of multi-coloured pens, pencils, slide rules, and so on, in his pockets." These traits he continued through his lifetime.

Arthur Raymond, Vice-President for Engineering of Douglas Aircraft, and Wilbur Wright Lecturer, has written to me to say:

> "He would continually multiply and divide numbers in his head in class while the class feverishly checked him on the slide rule and he would invariably be right to the third decimal. In spite of his youth, he had no difficulty in demonstrating his authority and superior knowledge in comparison with those he was teaching."

In concluding these few notes on Ed Warner as a teacher, I quote from another M.I.T. student of his, F. T. Kurt, now employed at Grumman Aircraft Company:

> "My first meeting with Professor Warner—it was not until years later that we could call him 'Ed'—was in 1923. He was the first man I had met with two telephones on his desk. The interview pertained to my registering at Tech and lasted for some time. Somehow, he managed to give me his complete and apparently uninterrupted attention while answering phone calls, some of which came two at a time, and other queries from his two secretaries. He would pass me a brochure and turn to his dictaphone while I read. When I looked up, he would stop in the middle of a sentence, and after further conversation with me, he would, at the next break, resume dictation precisely in the middle of the interrupted sentence! He was a dynamo of energy.
>
> "Ed was a wizard in mental arithmetic. When giving a lecture, he not only put on the board the most complicated differential equations, introducing a host of constants from memory, but then solved them showing all computations, as fast as he could write. Later, at home, we would laboriously trace through our notes and find that his answers were precisely accurate. He could actually multiply, say, $4{,}327 \times 7{,}851$ immediately in his mind. A group of us asked him to give a series of six lectures on his methods. Eighty showed up for the first, and no one followed him more than ten minutes. Twenty appeared for the second, our original group only for the third, and we then called it off!"

Scientist

Dr. Warner's scientific activities can perhaps be said to have commenced with the designing of the prize-winning glider in 1911. Surely, such a feat at that time must have entailed considerable delving into aeronautical science as well as engineering. But it really started in earnest early in his term at M.I.T. when he conducted wind tunnel experiments in a 4 ft. open circuit tunnel designed by his teacher and life-long friend, Jerome Hunsaker, former Chairman of the N.A.C.A. and Wilbur Wright Lecturer.

His scientific competence was demonstrated by the many technical articles he prepared and the books on aeroplane design he wrote. But I feel that possibly it was best shown by his work with the N.A.C.A. Dr. Hugh Dryden kindly furnished the following résumé of these:

> "In the records of the National Advisory Committee for Aeronautics, Warner's name first appears as one of the authors of Technical Report No. 17, 'An Investigation of the Elements which Contribute to Statical and Dynamical Stability and of the Effects of Variation in Those Elements.' He was then an assistant in aeronautical engineering at the Massachusetts Institute of Technology.
>
> "In January 1919, John Victory, Executive Secretary of N.A.C.A., visiting the Institute to recruit a technical staff for the new N.A.C.A. Langley Laboratory, 'discovered' Warner and hired him on the spot to start N.A.C.A. research at Langley. He was chief physicist, first head of aerodynamic research, first chief of flight research, and designed and supervised the construction of the first wind tunnel built by the N.A.C.A. Technical Reports 73 and 78 describe his work on the design of wind tunnels and wind tunnel propellers. Other reports and notes dealt with wind tunnel balances, accelerometers, stability and control, stress analysis, slip stream corrections, and the problems of the helicopter.
>
> "After his return to the Massachusetts Institute of Technology as associate professor of aeronautical engineering in 1920, Warner spent the summer of 1920 as technical attaché for N.A.C.A. in Europe. He also retained membership on the N.A.C.A. Committee on Aerodynamics from 1919 to 1945, serving as secretary from 1921 to 1925, and as Chairman 1935 to 1945. He was a member of the National Advisory Committee for Aeronautics and of its Executive Committee for the period 1929 to 1945. Warner also served the N.A.C.A. as Chairman of the Committee on Aircraft Accidents from 1933 to 1941, Chairman of the Committee on Operating Problems from 1942 to 1945, and member of the Committee on Materials for Aircraft from 1920 to 1945.
>
> "The series of Technical Memoranda of the N.A.C.A. during 1922 and 1923 contain reprints of some 45 articles on a diversity of aeronautical subjects which were originally published in the *Christian Science Monitor*. A few typical titles, illustrating the great diversity of Warner's interest, are as follows: Giant Airplanes, Monoplanes or Biplanes, Airplane Speeds of the Future, Prospects of Helicopters, Future of Airship, Factors of Safety, Metal Construction of Aircraft, Depreciation of Aircraft, Night Flying, Prevention of Fire in the Air, Comfort in Flight, Subsidies for Air Transport, Police Regulation of Aircraft. His judgment and foresight are well illustrated by the following statement made in December 1922: 'On the whole, it seems perfectly safe to say that we shall not see the attainment of a speed in excess of 325 m.p.h. within the next ten years unless by some unlooked-for invention which modifies the very fundamentals on which the operation of aircraft now rests.' Actually, this mark was exceeded by the British Supermarine seaplane in 1929 and by Howard Hughes' racer landplane in 1935."

Although at about midway in his career his scientific endeavours were to some extent eclipsed by his preoccupation with economic and operational aspects of air transportation, he continued his keen interest and from time to time to the end, made important contributions, recognised and respected by his scientific associates. L. Welch Pogue, Chairman of the Civil Aeronautics Board at the time when Warner was Vice-Chairman, wrote me the following:

> "I certainly do share your grief in the passing of Ed Warner. I was at the opening session of the Triennial Inspection of the Ames Laboratory of N.A.C.A. at Moffett Field, California, when John Victory, as one of the first orders of business, gave us the sad news. The entire 480 representatives of aeronautical interests stood for a time in silent tribute to the memory of this great man."

Miscellaneous Activities in the Public Interest

Dr. Warner responded willingly to requests to serve on government commissions or other public assignments throughout his lifetime. This also was true for his active participation in the work of many technical societies and associations, both within and outside of aeronautics. In fact, practically nothing that he undertook was other than an activity closely associated with the public interest. This holds true as regards his participation in all of the main subdivisions into which I have divided this article, but here, I will mention a number of the miscellaneous undertakings that punctuated the major tasks that he set for himself.

Just after the First World War, he wrote many articles on air policy including the drafting of aeronautical legislation for the State of Massachusetts which was adopted by the legislature in October 1920. He also worked closely with officials of Boston on aviation matters and served as Chairman of the Massachusetts Advisory Board on Aeronautics at about that time.

In 1924 and 1925, he was a consultant for the U.S. Air Mail Service (then operated by the Post Office Department) on matters relating to equipment and on air navigational problems. During this time, he was a passenger at the time of a crash of a Ford aeroplane. Many years later, he wrote to an acquaintance, "I

Courtesy of the I.C.A.O. Bulletin.

Edward Warner (*right*) with Professor Alexander Klemin (*left*) and G. Marshall Denkinger (*centre*) outside the M.I.T. wind tunnel in 1917. Warner and Denkinger were assistants to Professor Klemin.

informed the Second Assistant Postmaster General that the aeroplane had shown admirable structural characteristics and that in my opinion, a similar crash of an aircraft of conventional wooden structure would have caused some death or serious injuries."

In 1925, Warner was selected as President of the Boston Airport Corporation, as mentioned heretofore.

During the same year, he was selected as consultant for the President's Aircraft Board of which Mr. Dwight Morrow was Chairman. He reported to the Board the results of studies of existing and proposed aeronautical developments in foreign countries. By virtue of the recommendations of the Morrow Board, aviation got a considerable boost in the United States as evidenced by Congressional legislation establishing an Assistant Secretary for Air in the Department of Commerce, Army and Navy.

Warner was appointed to this Navy post in 1926 where he served as Assistant Secretary of Navy for Aeronautics until 1929. His secretary, Miss Brown, writes: "I recall when in Washington, how furious Admiral Moffett used to get, as he said he was not accustomed to being talked to as a student. His Aides also had their difficult moments," resulting from Ed's professional manner.

Dr. Hunsaker has written to me about this period in Ed's career at which time there appeared his interest in policy-making as distinguished from, and eventually to overshadow to some extent, his scientific and technological interests. Hunsaker writes, "Now there is no fight between ship men and air men. Naval aviation is the striking arm of the fleet, its scouting arm and its A.S.W. weapon. Naval aviation is now really in control everywhere. Ed Warner helped to start the momentum. He had to support aeronautical budgets, see that aviators did not fail of promotion because of 'seagoing' records, and so on. In all this he was very helpful."

Dr. Warner had always taken an active interest in the work of technical societies and during the 1920's his services to the Society of Automotive Engineers were myriad. He had served as Chairman of the Publications Committee; of the Aeronautical Division of the Standards Committee; of the Wright Brothers Medal Committee; of the Constitution Committee; of the Overseas Relations Committee; of the Students Committee and other technical and administration S.A.E. groups. In 1930, he was made President of the Society.

At about this time, it was becoming increasingly evident that there should be an exclusively aeronautical society in the United States. As early as 1928, Dr. Hunsaker and Major Lester Gardner had had correspondence relevant to this need with Captain Pritchard and Lord Sempill, Secretary and President respectively, of the Royal Aeronautical Society. The decision to proceed with a U.S. society patterned on the R.Ae.S. was made in 1932 and Warner was one of the group involved in the preliminary discussions. Lester Gardner stated the following in his "History of the Institute of the Aeronautical Sciences" on the occasion of its 10th anniversary in 1942: "Mr. Warner supplied the next important item—the name of the new organization. It was he who suggested 'Institute of the Aeronautical Sciences,' which met with the instant approval of the organizers." Dr. Warner played a leading role in the I.A.S. throughout his life.

In 1934-35, when the air mail difficulties were plaguing the country, President Roosevelt appointed the Federal Aviation Commission to explore the many vexing problems involved. Edward Warner was made Vice-Chairman. There is wide speculation that it was he who drafted much of the resulting report.

A little later, in 1938, Warner performed a quasi-public service in developing, with others, a means of determining appropriate compensation for mutual use of patents held by members of the Manufacturers Aircraft Association. His association with the Civil Aeronautics Authority started at about this time.

Then, in 1941, he was appointed a member of the staff as special consultant of the so-called Harriman Commission which served in London on a multitude of problems relating to the war effort, including Lend-Lease agreements. This was an assignment much to his liking. He felt he was making a real contribution to the war effort of the allied nations in whose cause he felt so deeply. I know he was glad to be in London at the time of major bombing attack by the Germans.

Author and Editor

Throughout his career, Ed Warner was a prolific writer and speaker. It will be recalled that in the early 1920's, he wrote an extensive series of articles on a variety of aviation topics for the *Christian Science Monitor* of Boston, later published by the N.A.C.A. as *Technical Memoranda*. His first book, *Aerostatics*, appeared in 1927 followed by the volume, *Aeroplane Design—Aerodynamics*. This rather classic text served as a "bible" in this field for many of us. Many observations in it were ahead of the times. In subsequent years, what appeared to me as a new concept, would upon search, be found in *Aerodynamics* worked out in detail but not fully appreciated nor absorbed on originally reading.

From 1929 to 1934, Dr. Warner was Editor of *Aviation Magazine*, the leading publication in the field

at that time. This McGraw-Hill magazine achieved a high standing during his encumbency as Editor. During this time, in collaboration with Paul Johnston, the *Aircraft Handbook* was prepared and published.

Then in 1936, under the title *Aeroplane Design—Performance*, the 1927 text was revised and expanded to result in an entirely new book. This volume contained many methods for rapid determination of aircraft performance characteristics most useful to the aeroplane designer and engineer. The use of the so-called "Warner K" speed formula proved of great value to me during the war in judging speed of foreign aircraft with only power and wing area available.

In 1937 and 1938, Dr. Warner gave the James Jackson Cabot lectures at Norwich University. These were published in expanded form in 1938 and served many of us as valuable and quotable material in our own work, whether in preparation of talks, in engineering analyses, or in making air transportation forecasts. The titles were, "The Early History of Air Transportation" and "Technical Development and its Effect on Air Transportation."

One might, at this point, appropriately mention Ed's great ability as a speaker. His addresses were always noteworthy for their logical development and their beautiful English. Sometimes the sentence structure might become complex, but each would wind up perfectly from the standpoint of syntax. (His listeners would often amusedly wonder whether or not he would extricate himself from a maze of consecutive dependent clauses, but he always did.)

Another trait related to the above and to his speaking abilities, whether from a platform or in private conversations, was his great ability to concentrate and to hold off all distractions while so engaged. I am sure others experienced my own observation of his ability during a conversation to conclude the thought he might be dealing with before smilingly deferring to his listener, and with rapt attention await the response.

A list of all his speeches and articles would indeed be a lengthy document and will not be attempted, but I would like to mention one, the thirty-first Wilbur Wright Memorial Lecture read in London in 1943 entitled "Post-War Transport Aircraft." The following is quoted from that lecture:

> "I am concerned here with economics and with engineering; and I shall say nothing of politics except to make a personal profession of faith, in which I hope you can all join. My hopes for the future are rooted in the central hope that air navigation agreements will henceforth be drawn to the presumption that air transportation is a good thing; that the need of the whole world to share in its benefits should be a prime consideration in the planning of air routes; and that there shall be no return to those evil days when air transport was regarded with such caution and suspicion that it had to be administered in paltry and carefully measured doses, with the authorization to operate internationally being parsimoniously doled out, schedule by schedule."

Relevant to this lecture in general, Laurence Pritchard writes, "Here was authority and experience speaking with a confidence which was refreshing in the amount of information Warner gave, speaking too, clearly, at a speed only slightly less than that of sound!"

Consulting Engineer

From 1936 to 1939, Dr. Warner took up temporary residence in California as a consulting engineer to United Airlines which, in association with other U.S. airlines, was interested in the development of a four-engined transport aircraft. His initial work on specifications was extended to the drafting of a contract for the development and construction, awarded to Douglas Aircraft Company. He worked with Arthur Raymond in establishing the design requirements of the DC-4E aeroplane, a predecessor of the later considerably altered DC-4 and war-time C54 aircraft. A scientifically important accomplishment was the conversion to engineering language of the flying qualities desirable or necessary for a transport aircraft as determined from the descriptions of pilots. Concerning the contract and subsequent development of the aeroplane, Arthur Raymond writes:

> "The contract itself was a very complicated document, but although there were bona fide lawyers who had the responsibility of preparing it, a lot of the wording was in actuality worked out by Ed who, as you know, had a remarkable mind—one that was able to phrase things in legal fashion and actually think in a way that lawyers think. Ed kept in close touch with the design of the airplane as it went along and actually participated in some of the flights for the purpose of observing stalling and other characteristics at first hand. His contribution to that project was considerable."

U.S. Public Servant

Although Dr. Warner carried out all of his multitudinous activities in the interest of the public it was not until 1939 that he accepted a position in government that made him a public servant. The agency of government was the Civil Aeronautics Authority created by the Act of Congress known as the Civil Aeronautics Act of 1938.

Because of his technical background as well as vast experience, Dr. Warner was called upon to do a large share of the work in formulating regulations dealing with certification of airmen, of aircraft, and of flight operations. This did not, however, diminish his interest in or contributions to those functions of the Board that dealt with economic determinations. Dr. Warner always evinced a particular interest in the simplification of safety regulations, especially those affecting personal flying, and in the continued rationalisation of airworthiness requirements and operating practices.

It seems most appropriate to describe his activities in C.A.B. through the words of his close associate, L. Welch Pogue, who was General Counsel for economic regulations of the Board from the time it was created until 1941, then General Counsel to January 1942, when he was made Chairman. Mr. Pogue has written the following:

"Almost immediately upon the establishment of the Civil Aeronautics Authority in September 1938, Ed Warner was called in as a voluntary adviser and then as a consultant to the Authority. Although an aeronautical engineer with a worldwide reputation, his work for the Authority at that time was largely in the economic regulatory field, particularly with respect to the ever-perplexing problems of air mail rates. His contributions to the Authority were so outstanding and his many other qualifications were so good that he was appointed to the very first vacancy which occurred on the Authority.

"He became a member of the Authority on May 5, 1939, and served until he resigned on September 20, 1945, to become President of the International Civil Aviation Organization in Montreal. He was Vice-Chairman during two periods—from July 11, 1940 to December 31, 1941, and from December 18, 1942 until the time of his resignation.

"In 1944, at the important Chicago International Conference on Civil Aviation, Ed Warner was a key figure. Respected as he was by all representatives of foreign countries accredited to that conference, he was efficiently and excessively used as an emissary and contact man, particularly with the British and the Canadians with whom the United States, during the conference, had the most occasion to settle important issues affecting the conference. He had the happy faculty of combining idealism with a recognition of the practicable, and therefore, seldom reached for the stars when he knew they were unobtainable.

"He was a natural trouble-shooter for the five-member Authority (whose name was officially changed in 1940 to the Civil Aeronautics Board). He was sent on many missions to the State Department, to the War Department, to the Department of the Navy (with whom he had served before), to Alaska, to the Caribbean, and to meetings with the regulated industry, particularly where safety or engineering problems were involved.

"He was a prodigious worker. He never wasted a minute; he dictated into a dictaphone at home in the evening; in the middle of the night when he happened to wake up; and in the morning. As an illustration of his prodigious capacity to work, one of the members of the Board during the time when Ed Warner served and who was very conscientious about reading all memoranda on issues presented, frequently referred to analytical data coming from Ed Warner's typewriter as 'descending upon me like snow from Heaven.'

"He also had the capacity to maintain a delightful sense of humor and good relations toward everyone with whom he worked. Though intolerant of mediocrity on an intellectual basis, he was always kindly toward everyone on a personal and emotional basis. He detested indirection."

In connection with his work on the C.A.B. but as well in many other undertakings, Ed Warner had a deep and abiding interest in safety of flight. My own preoccupation with all phases of safety brought me in close contact with him in this field. But I will present an appraisal of him from this standpoint through the words of our mutual friend, Jerome Lederer, Director of the Flight Safety Foundation and the Cornell-Guggenheim Aviation Safety Center and who has made aviation safety his life work and is so eminently qualified to speak. He writes:

"It is difficult to find any activity of Ed's which was not associated with safety. In 1922, long before there was any governmental activity in the field, he was instrumental in developing an Aviation Code sponsored by the S.A.E. At meetings of technical societies, one could always count on an authoritative discussion, invariably tied to safety, led by Ed.

"His fervent inclination for achieving safety was always along lines of education and motivation rather than regulation or punitive measures. He also felt strongly that the individual should be permitted to do what he pleased insofar as personal danger was concerned so long as the action did not unreasonably jeopardize the safety of others. However, he was the principal factor in the promulgation of what is now known as CAR 4B, the U.S. Government airworthiness regulations governing the design of transport aircraft, which served as a world model.

"It was Ed who brought me to the C.A.B. as Director of the Safety Bureau. My staff and I always relied on him for advice and ideas in connection with the analysis of accidents and the promulgation of regulations. His many penetrating suggestions made him an immense influence in safety, and a catalytic agent in stimulating air safety research.

"I.C.A.O., of course, is the permanent monument to Ed's tremendous energy, statesmanship, executive ability and knowledge. I was honored to serve under his Chairmanship as a member of I.C.A.O.'s Jet Implementation Panel, which was the last assignment prior to his retirement from I.C.A.O.

"Ed must have turned down a great many offers after his retirement; the fact that he accepted membership on the Board of Governors of the Flight Safety Foundation and was active in its affairs is indicative of his abiding interest in safety.

"For aviation to go on without Ed Warner seems incredible, almost impossible. His influence and philosophy were such that his ideas and devotion will continue to be felt indefinitely.

"In 1956, the Flight Safety Foundation presented Ed with its award for distinguished service in achieving safer utilization of aircraft."

I cannot refrain from commenting additionally, by way of emphasis, on what Mr. Pogue has said concerning Warner's activities at the Chicago conference. As Technical Secretary of the Conference, I had the privilege of discussing many problems relating to the *Technical Annexes* of the Convention with him. I am therefore sure it is no over-statement to say that without the availability of his great knowledge and his freely given advice and guidance, the Annexes would never

Edward P. Warner, as Assistant Secretary of the Navy for Aeronautics, being helped into a parachute before flying out to the aircraft carrier *Langley* in 1928.

Courtesy of the I.C.A.O. Bulletin.

have been the outstanding documents that they were and as well, that the other successes of the Conference would have been far less than they were. The fine start in all technical matters given by the Annexes has contributed much to the tremendous growth of international air transportation.

Dr. Warner was the U.S. Delegate to P.I.C.A.O., the Provisional International Civil Aviation Organization where he performed yeoman service in all initial work necessary to the smooth transfer of its activities to the permanent organisation, I.C.A.O.

International Public Servant

Dr. Warner's illustrious career in aviation was crowned by his twelve years of achievement as President of the Council of the International Civil Aviation Organization and its provisional predecessor, P.I.C.A.O. These organisations stemmed from the Chicago Conference of 1944 and the Interim Agreement and Convention eminating from it. It is well to note in Dr. Warner's own words his concept of the organisation's basic aims and as well, his philosophy of the necessary method of conducting international affairs.

As to *modus operandi*, I quote two passages taken from his article in *Foreign Affairs*, a U.S. quarterly, for April 1945:

> "From the experience gained at Chicago, we shall have to go forward pragmatically, in search of a solution which no one will find ideal, but which all can accept with reasonable satisfaction. Collective ingenuity may discover a principle for universal application, and collective goodwill may win acceptance for it. Alternatively, there may be a large number of separate solutions, not identical, yet with some features in common, defining the terms of commercial association between particular pairs of nations."
>
> "Agreement is not to be found through mutual recriminations but in an honest and continued effort to understand one another's positions and problems and to discover new solutions that will approach the requirements of all parties."

And on fundamental objectives of I.C.A.O., Dr. Warner wrote the following in another American quarterly, *Air Affairs*, for the spring of 1950:

> "It exists primarily to encourage civil aviation and to encourage good and friendly international relations through civil aviation. Its primary reason for being is to make it possible to fly more easily, more safely, more economically, and more pleasantly, with the least possible obstruction by international boundaries and the best possible assurance that those wishing to engage in international air transport will have a fair opportunity to do so, whatever their nation may be, and that the entry of foreign aircraft will be a benefit to the states that they enter and not a hazard. I.C.A.O. should be judged, now and in the future, by its success in promoting these ends."

How successfully Ed worked and achieved agreements in the spirit of these precepts!

Dr. Warner, as the U.S. delegate, attended the very first meeting of the Council of P.I.C.A.O. on 15th August 1945 and was elected its President. He immediately resigned his membership on the U.S. Civil Aeronautics Board and broke off all other connections of a purely American character. When I.C.A.O. was established as a result of the ratification of the Convention and International Agreement by the requisite number of nations, Dr. Warner was elected President of its Council (28th May 1947) and was re-elected successively in 1950 and 1953. By the time of a new election for the 1957 term, he had decided to relinquish the helm and with Dr. Warner's strong endorsement, Dr. Walter Binaghi was elected as his successor. Warner's work on the Council as presiding officer represented but a small part of the job as he conceived it. He regularly attended meetings of the many subordinate bodies where the benefit of his advice was invaluable. These covered a wide range of functions such as technical, economic, legal and administrative. The extensive programme of regional meetings took him to all parts of the world since he conceived it to be his duty as well as his desire to attend them all. As Sir William Hildred, President of I.A.T.A., the International Air Transport Association, has said, "He had carried his briefcase and his typewriter a million miles during the twelve years of his presidency."

I have headed this section "International Public Servant." This Warner surely was. He was meticulous

in divorcing himself from any implications of national interest, a determination that, with his other high qualifications, made him ideal for the post of President of I.C.A.O. There follow four quotations from his close associates in I.C.A.O. to bear this out.

Admiral Paul Smith, U.S. delegate to I.C.A.O. for eight years and a close associate at Chicago and before, has written to me:

> "Ed's steadfast devotion to the highest principles of impartiality in this post caused him some difficulties which might have seemed amusing to others who did not know him so well.
>
> "I remember one time when his American passport had expired, he came to me, the U.S. Representative, sincerely troubled over how he could swear the oath then required for a United States passport without prejudice to his over-riding responsibilities as a high international public servant."

And again, Dr. F. H. Copes van Hasselt, Netherlands representative to I.C.A.O., in an appreciation appearing in the I.C.A.O. Bulletin in 1957 at the time of Dr. Warner's retirement said, "He was an international civil servant in the true sense of the word." And Sir Frederick Tymms, U.K. delegate to the Chicago Conference, to P.I.C.A.O. and I.C.A.O., wrote in the London *Times* of 21st July 1958:

> "It was my privilege to be closely associated with him at that conference and for the next ten years. Such was his pre-eminent fitness for the great responsibilities of the post that while Dr. Warner was prepared to serve, no other candidate was considered or even proposed at any of the succeeding elections of the President, until his final retirement in 1957. He was, by nature, an 'internationalist' and no man could have held the scales more evenly than he in the many difficult debates and conflicts of interest which are perhaps inseparable from an international organization."

In addition to his qualifications as an international public servant, there were, of course, many other qualities from which his power in conducting the affairs of the Council stemmed. Under the title, "The Power of a President," Henri Bouché, representative of France and a lifelong friend of Warner's and close associate at Chicago and thereafter, defines Warner's qualities in this regard. I quote from the 1957 *I.C.A.O. Bulletin* previously referenced:

> "His power derived first from his vast knowledge. I have never met *anyone* who, in as much as I am able to judge, had such a grasp as Dr. Warner has of all those sciences, techniques and practices that constitute the art of aeronautics, or of all those other fields that, although outside the boundaries of aviation, nevertheless cannot be ignored when one must plan for, and place aviation in our civilization.
>
> "His power also resulted from his extraordinary passion and capacity for work.
>
> "His strength was due also in large measure to his infinite caution in human contacts, and to his ever-present desire to avoid hurting anyone. All of us have noted his absolutely equal respect for all representatives on Council, however different in age, career, working methods and tempo of contribution to the common effort, however great, at times, the distance between what they were and what he was. In his estimation, each and every one of those men deserved equal respect, and he also respected in them the States which they represented.
>
> "He also gathered strength from his constant, and truly deserving efforts to understand the viewpoints and positions of others.
>
> "Finally, his strength derived from his unrelenting search for unanimity, or for substantial majorities, lacking which he feared that this adolescent international organization might be exposed to too many internal tensions, or to too many attacks from outside. How many thousands of miles did he not walk in twelve years, in quest of this unanimity, along the corridors of our successive Montreal headquarters, his strong shoulders only slightly stooped as he emerged from an office where the desire for conciliation perhaps had not matched his own?"

Van Hasselt says, "Power may be imposed upon a man but influence emanates from himself, and this is the secret of Warner's success." And in the same Bulletin, Dr. E. M. Loaeza, representative from Mexico, in his appreciation entitled "A Man of Our Times," wrote:

> "My first impression of Dr. Warner as a technological man was that this aspect of his personality appeared to absorb him completely. However, as I grew to know him better over the years, I discovered in him a firm intention to give spiritual values the place they deserve, and a conscious effort to be guided in the conduct of his life by the search for understanding."

Among all the comments of commendation one might expect some criticism of him to appear. And yet, few, if any, do unless one counts an occasional reference to his excessive modesty. Admiral Smith wrote to me:

> "In presiding over the Council, he never protested any of the few unjust criticisms which were levelled at him, usually answered them patiently and almost apologetically, but always so factually perfect that his accuser usually ended up by apologizing for his impulsive action."

And as for any criticism by him of others, Van Hasselt has this to say:

> "To preside over a body consisting of persons of different cultures, and different languages is no easy task, yet the word 'criticism' is non-existent in his vocabulary, and never have I heard him say an unkind word with respect to somebody else. He always sees the good side of persons and things."

To illustrate Dr. Warner's great contributions to I.C.A.O. and to world aviation, I have selected two paragraphs. The *Bulletin* cited before in its own assessment, says:

> "In all these fields of I.C.A.O.'s endeavours Dr. Warner's personal contribution has been widely

acknowledged ever since the beginning of I.C.A.O.'s existence."

And Henri Bouché:

"During his twelve years of office as President of the I.C.A.O. Council, Dr. Warner has contributed to international civil aviation more than any other man could have done in his time and in his place."

Warner himself illustrates some accomplishments, not of himself, but of I.C.A.O. itself in the promotion of worldwide civil air transportation in a paragraph from his *Air Affairs* article written in 1950:

"Aircraft over most of the surface of the earth are already flown in accordance with I.C.A.O.'s Rules of the Air. Their personnel are beginning to be licensed to I.C.A.O.'s standards. Their pilots are briefed before flight by the use of N.O.T.A.M.'s (Notices to Airmen) prepared and transmitted in accordance with an I.C.A.O.-shaped pattern. The meteorological forecasts given them are drawn to an I.C.A.O. specification. After the flight begins, communication between the aircraft and the ground is conducted in I.C.A.O.-adopted codes and in I.C.A.O.-specified phraseology. The pilot makes his instrument approach, if an instrument approach be necessary, on an I.L.S. of which the characteristics have now been standardized by I.C.A.O. to assure the closest possible uniformity of approach path and of character of instrumental indication at aerodromes throughout the world. The charts of the country over which he flies, and the larger-scale chart that shows the surroundings of the airport at which he is to land and the standard method of approaching it, have their size, color scheme and the meaning of their symbols determined by an I.C.A.O. standard."

And now to give the more personal side of his dealings with his I.C.A.O. associates, I quote a few more passages. Van Hasselt has written:

"Despite all these activities Warner has always found time to show his keen interest in everything that concerned representatives more personally. Wherever he could help with advice, introductions, etc., he did so at his own initiative; whenever, with his wide reading, he came across an interesting publication regarding our countries, he would send it to us or call our attention to it.

"Although not in the least striving for outward show, Warner enjoys an immense popularity."

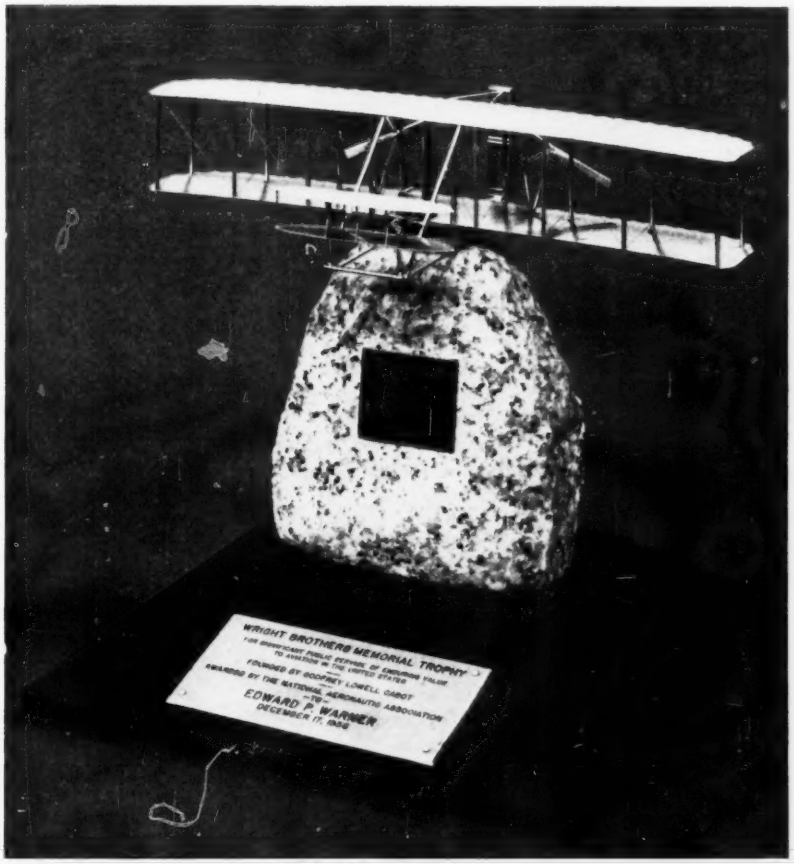
And Tymms:

"Many of the problems of international civil aviation have been guided to a surer and a better solution by his selfless devotion to his task. Yet, with infinite patience, he never failed to give the fullest opportunity for the expression of every varying shade of opinion.

"No one of those who have sat in the Council of I.C.A.O. under his Presidency, or of the many from all parts of the world who have gathered under his leadership at innumerable meetings, but owes much to his inspiration and counsel. Though devotion to his task was his all-pervading characteristic, all who knew him were vividly aware of the depth of human sympathy in him. Kindliness and a rooted disinclination to condemn were, in him, almost a fault. The world of aviation is the poorer for the death of one of its greatest figures."

In mourning Dr. Warner's death, I.C.A.O., in its official release, wrote:

"It is almost impossible to believe that his brain and his heart are no more active and working in that extraordinary rhythm of which only he was capable and, thanks to which, I.C.A.O. accomplished so much in such a short time. This



The Wright Brothers Memorial Trophy "For significant public service of enduring value to aviation in the United States" awarded to Dr. Edward P. Warner on 17th December 1956.

Courtesy of the I.C.A.O. Bulletin.

Organization is indebted to him for ever. Twelve years of his rich life were left for us and for international aviation. No one can read into any of our specifications, recommendations, studies or records, without constantly finding his thoughts and influence.

"To all who had the privilege of knowing him, he imparted his knowledge, his equanimity, his international spirit. In his personal contacts, he always gave warm friendship and kindness, which were particularly impressive because they came from an outstanding man, yet were wrapped in modesty."

The Man

The foregoing discussion and appreciation of Ed Warner's career brings out those of his traits of character as well as technical abilities that made possible his great achievements over so wide a range in aeronautics. For a complete picture of the man, one must recognise that his activities were not restricted to aviation but encompassed many other areas. He wrote to me in 1947, "There are many matters about which I would like to talk with you—not all of them by any means, limited to civil aviation." Paul Johnston writes of how he was always much intrigued by the theatre and attended performances whenever he could.

In the field of economics in all the ramifications that the word implies, his interest was keen and his observations were profound. He was an active participant in the meetings of the National Policy Committee in Washington whose objectives were to get together groups of responsible citizens whose common desire and purpose was to work toward the development of governmental policies by democratic means on a basis of general interest rather than special interests. Ed was instrumental in getting me to participate. This was just before and during the Second World War. The subjects discussed were such as the constitution, labour, economic policy, housing, relief, steps toward a durable peace, education, manpower, sundry problems related to winning the war, and a host of others. In all, Ed contributed much.

Then, too, Ed was an active participant in the Council for Foreign Relations, an influential group headquartered in New York that deals with a wide range of matters of public interest and which serves as adviser to the State Department in many assignments.

In conclusion, I will list those qualities of character and mind that made Ed such an outstanding man. These are my own assessments, reinforced by quotations appearing heretofore and a few hereunder:

Integrity—Welch Pogue writes:

"Ed Warner was a great man. He was able, but he also had that latch key to success—persistence; he was productive, versatile, friendly and kindly in disposition, and above all else, he had integrity. His shining armour of fidelity to his trust never, to my knowledge, was dimmed by a single stain. You always knew that he stood for what he believed to be right. What a giant in character he was!"

Courtesy of the I.C.A.O. Bulletin

Dr. Warner, as a delegate of the United States, at the first session of the Interim Council of P.I.C.A.O.

Industry—He was unsparing to himself in the conduct of all his many undertakings.

Encyclopaedic mind—His memory was prodigious. One was always amazed at the knowledge he possessed and the cataloguing of facts in his brain for almost instant availability when an appropriate occasion might arrive.

Technical brilliance—The range of his writing, books and speeches, is a testimonial to his technical knowledge and its application.

Concentration—He could concentrate on the matter at hand so completely as to cast out all others from his mind.

Modesty—Throughout his career, his associates have noted and commented on this characteristic. Henri Bouché wrote, "He over-estimated the external resistance, and under-estimated his own strength. Thus, his sin, if any, was too much modesty."

Understanding and sympathic nature—I am particularly aware of these qualities from my own correspondence with him over a period of thirty years. He always understood, usually over-praised and was invariably sympathetic. Laurence Pritchard writes:

"Neither of us can find words great enough to praise him for all he was. He made no visit of any kind to England without making a point of seeing us and spending some of his precious time with us. From the first day I met him, now just on thirty years ago, I found in him an astonishing understanding or sympathy."

Selflessness—Ed always thought of the other fellow's feelings when making statements. And particularly, he demonstrated this quality in his propensity for public service rather than personal aggrandisement or self-seeking aims.

International-mindedness—In acknowledging my congratulations on the occasion of the award to him of the Wright Brothers Memorial Trophy in 1956, he wrote to me:

> "The people who determine the award certainly have been most generous to me; but one thing that particularly pleases me about it, and that I am sure must please you too, is the implied recognition that the work in which I have been engaged for the last eleven years, in seeking to serve the whole world, performs a useful service also for each individual country."

Friendliness—Ed's friends and admirers were legion. His friendships were worldwide in range, but particularly I would stress those he made through the years in England. Pritchard has listed for me many British friends, but by no means all. After doing so, he says:

> "In a confidential note I wrote about him in 1943, I wrote, 'he is probably one of the most influential of the technical men advising the U.S.A. Government, and is looked upon in this country (*i.e.* Britain) as being the most knowledgeable and a first-class friend."

Kurt, his former student, writes the following:

> "It is perhaps a tribute also that my notes have dealt with Ed as a man and revered friend rather than to emphasize his international stature and widely acknowledged achievements. We all admired him for his terrific accomplishments, and I am sure you will receive abundant testimonial on these, but it is a personal friend we have all lost."

Dr. Warner was the recipient of many honours and awards, including medals and decorations from thirteen foreign countries. He received Honorary Fellowship from the Aeronautical Society of India, the Canadian Aeronautical Institute and the Royal Aeronautical Society. At home, he was awarded Honorary Fellowship in the Institute of Aeronautical Sciences and received the Wright Brothers Medal, the Daniel Guggenheim Medal for Achievement in Aeronautics, the Wright Brothers Memorial Trophy. He was also the recipient of the F.A.I. Gold Medal.

I close with three quotations:

Paul Johnston, Director of the I.A.S.:

> "My own appraisal of Ed as a person and as a scientist and teacher will, of course, be considerably biased because I consider that, except for my own father, Ed Warner contributed more to whatever my career has been than any other person. As far as I am concerned, there has always been Ed Warner in the background and I have great difficulty in making myself believe that he is gone."

Laurence Pritchard, long-term Secretary of the R.Ae.S., wrote in *The Aeroplane*, 25th July 1958:

> "It has been my very good and great fortune to have known Ed Warner for thirty years, both in his home and public lives; years of great achievements and distresses both for this country and his own. He served both countries well, with all his heart, all his faith in their common destiny, all his unconquerable energy."

Stanley Krzyczkowski, Technical Director of I.A.T.A. said of him:

> "A big heart coupled to a big brain."

Some Design Problems in Inertia Navigation*

by

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(Deputy Controller (Aircraft Research and Development), Ministry of Supply)

1. Introduction

Until the past few years inertia navigation has been a little known subject due to a cloak of security; but recently a number of informative reports and articles have been published, notably those by Adams[1], Schnerb[2] and Klass[3], all published during 1956. Moreover, many of the components of inertial systems have emerged from the early development stage, during which they were largely monitored by military agencies, and may now be purchased with a guaranteed performance specification.

The applications of these components are, however, still largely military in nature and it is difficult to produce an interesting paper, which does not merely repeat what has already been published nor transgress into those forbidden fields of military application.

While I thought it necessary to give a brief description of the fundamentals of inertia navigation, for the benefit of those who are new to the subject, my main aim is to describe, in greater detail than heretofore, some of the fundamental components of these systems, to deal with certain design problems which we have encountered in the U.K., and especially to describe and stress the necessity for special methods of testing such components; indeed, when dealing with equipment of the highest precision, the methods of proof and test may introduce problems of equal magnitude to those in the main development. In this connection, I would like to pay a tribute to the excellent and painstaking work, which has taken place in these fields, by the scientists and engineers of the Instrument Department of the Royal Aircraft Establishment and the research laboratories of the British Sperry Gyroscope Company. The methods of testing gyroscopes and accelerometers, which are described later, were devised by J. H. Whitaker and J. V. Carter, both of the Royal Aircraft Establishment.

1.1. GENERAL PRINCIPLE

The basic principle of inertia navigation is the determination of the position of a moving vehicle by means of inertia effects observed within the vehicle itself. To reduce the problem to its simplest terms, consider a body moving in a horizontal plane with only two degrees of freedom such that its movement can consist only of translation within the plane, with no rotational or vertical movements. It will be seen that measurements obtained from two accelerometers mounted along and perpendicular to an axis of reference within the body can define the movement of the body completely. The first integral of the accelerations will give the velocity vector at any instant and the second integral of the accelerations will give the displacement of the body, relative to the starting point, at the same instant. If the body is allowed to have rotational degrees of freedom, we see that the accelerometers must not be mounted along axes fixed to the body because they will no longer measure the accelerations within the plane when the body rotates and if the problem were complicated still further by introducing the force of gravity perpendicular to the plane, any tilting of the accelerometer axes will introduce a component of acceleration due to the gravitational force. Except possibly during a period of very large acceleration of the body the spurious accelerations due to the gravitational components are likely to outweigh considerably the horizontal accelerations and to introduce large errors for relatively small angles of tilt.

It is clearly essential, therefore, that within the body there should be maintained a platform which is always perpendicular to the gravitational force and the accelerometers must be mounted on this stabilised platform. We cannot, of course, maintain the stabilised platform by a level or a pendulum because this would be falsified by the accelerations to be measured. If its period is fairly short the platform would orient itself according to the apparent vertical, always remaining perpendicular to it and the accelerometers would not register any accelerations. We have to choose a platform which is insensitive to any acceleration or field of force and this can be provided by a gyroscopic system fixed in space. The platform is oriented before departure with great exactitude and provides a horizontal plane during the journey by preserving this orientation.

If we now consider the problem in relation to the earth itself, the situation changes considerably. It is now of no value (except for small displacements relative to which the earth may be considered as "flat") to maintain a platform, by means of gyroscopes, at a fixed orientation in space because the vertical itself rotates in space as the body moves over the surface. We are therefore faced with the problem of how to keep our platform always in a horizontal position as the body moves around the earth. We might perhaps consider a pendulum as an erector for the gyroscope but this is subject to the same errors as were discovered in considering a pendulum as a space reference on a flat earth and will not give the answer we require.

Further improvements in performance can only be obtained by feeding to the system more data. An

*A paper originally given in Paris in May 1957 at the Aeronautical Congress organised by O.N.E.R.A. Although this paper has been published by O.N.E.R.A. in the Proceedings of the Conference it is understood to have had a limited circulation only and is reprinted in the *Journal* as it is one of the comparatively few British papers available on this subject.

advance may be obtained by feeding in the assumption that the vehicle is always proceeding in the direction in which it is pointing, *i.e.* a centrifugal force is always associated with a corresponding change in the fore and aft line of the aircraft. There is a whole family of devices employing this principle, notably the Lanchester Pendulum and the Tilted Axis gyroscope.

The rotation of the aircraft is detected by a horizontal axis gyroscope (or by the component of angular momentum of another gyroscope on a horizontal plane). Suppose $I\omega$ is the angular momentum in the horizontal plane and $\dot{\theta}$ is the rate of turn then if the gyroscope is constrained to keep its axis horizontal it will generate a torque C given by $C = I\omega\dot{\theta}$.

It can be arranged that this exactly balances the centrifugal force which is proportional to $V\dot{\theta}$ where V is the speed of the vehicle. The necessary condition for the Lanchester family is therefore that the system has a component of angular momentum (*e.g.* a gyro speed) in the horizontal plane proportional to the speed of the vehicle. In the Lanchester pendulum the gyroscope is set transverse to the aircraft; in the tilted gyroscope the axis of a single gyroscope is tipped forward along the fore and aft axis and the speed relationship is obtained by varying the angle of tilt.

The chief snag in this type of system is that a small yawing movement of the vehicle (unassociated with a change of course) can be registered as a centrifugal force and produces a false vertical signal: this can be countered to some degree by constraining the system to the flight path rather than to the fore and aft axis.

This type of system has, however, many practical applications in such items of equipment as gyro bomb sights and flight instruments but its long term accuracy is not sufficient for navigational usage. The Lanchester pendulum is, however, of some historical interest since its inventor made one of the first steps towards the solution of the problem of obtaining a true vertical, around the turn of the century.

The next great step was an idea in 1923 by Schuler[1], which led to what is usually described as the Schuler pendulum. In simpler terms it is based on the principle that if one had a pendulum whose length is equal to the radius of the earth it would be independent of lateral accelerations and one would be able to move the point of suspension around the earth's surface without disturbing the bob of the pendulum in any way and the "string" of the pendulum would always indicate the vertical. Naturally we cannot construct a string pendulum of such a length but Schuler pointed out that the same effect is obtained with another physical system having the same period of oscillation. It is easy to calculate that the period of oscillation required is close to 84 mins., and for that reason the name "84 minute pendulum" is also used to describe the system.

To realise such a system in practice a gyro-stabilised platform is made to rotate through the same angle as the vertical is displaced due to the movement over the earth's surface. This is done by a double integration of the horizontal accelerations sensed by stabilised accelerometers and the result of the integrations fed to a torque motor which changes the orientation of the gyro system. A schematic arrangement of a Schuler gyro pendulum is shown in Fig. 1.

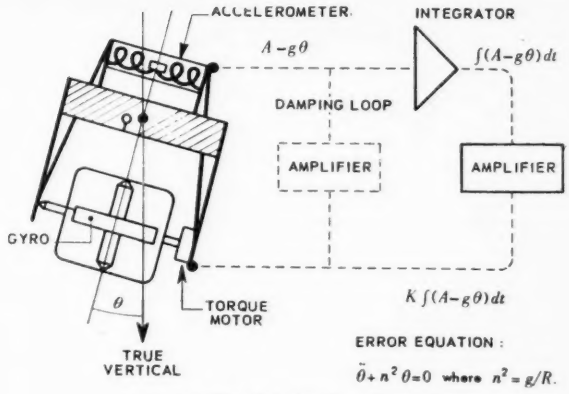

FIGURE 1. The Schuler Gyro Pendulum.

If θ is the instantaneous value of the angle between the true vertical and the reference vertical, it is also the angular measure of the error of such a system. The equation for the harmonic motion of θ can be written $d^2\theta/dt^2 + \alpha\theta = 0$ where α is constant. For Schuler tuning $\alpha = n^2 = g/R$.

For the system shown in Fig. 1 the error equation (neglecting the effect of earth's rotation) is

$$\frac{d^2\theta}{dt^2} + \frac{Kg\theta}{I\omega} = \frac{d^2\theta_0}{dt^2}\left(\frac{RK}{I\omega} - 1\right)$$

where K is an adjustable constant depending on the gain of the feed-back loop. $I\omega$ is the angular momentum of the gyroscope and θ_0 is the angular displacement of the platform along its path over the earth's surface. If we choose K to be equal to $I\omega/R$ the term on the right-hand side vanishes and the equation reduces to $d^2\theta/dt^2 + (g/R)\theta = 0$ which is the form we require.

There are complications due to the rotation of the earth and the fact that the radius of the earth is not constant but these can all be allowed for since they are readily predictable with the necessary accuracy.

We have seen that the realisation of an inertia navigation system depends on being able to obtain an accurate vertical reference in the moving vehicle and that once the vertical has been defined and the platform mounted perpendicular to it, the possibility of using accelerometers mounted on the platform to sense the horizontal accelerations and thence the horizontal displacement, is opened up.

Having once obtained a reliable vertical reference on the vehicle however, a second method of navigation becomes possible on a spherical earth which might be described as a "precessional" system. Since the Schuler pendulum has to be used to maintain the vertical it seems logical to measure the rotation of the platform which has been fed into the system in order to maintain the vertical and this measure of the rotation can be transformed into a position change on the surface of the earth.

Basically then there are two distinct systems of inertia navigation, both of which depend on a stabilised platform. In one case, which we might call an "accelerational system," we use the double integral of two horizontal accelerometers to give distances travelled along two directions at right angles. The gyro stabilised platform is levelled initially but no attempt is made to rotate it to allow for the curvature of the earth. It is therefore only applicable to very short flights over which the earth can be regarded as flat. In such a system a zero error of an accelerometer or tilt of the stabilising platform causes distance errors which increase as the square of the time. As an example, a zero error corresponding to a platform tilt of 1/10 degree will lead to a distance error of 68 miles after one hour.

In the other system, the "precessional" system which has just been described, a vertical gyro (compensated for earth rotation components) is maintained in the vertical by the application of controlling torques about two axes at right angles: the integral of the applied torques is proportional to the angle through which the gyro has been precessed and through which the vertical has turned relative to earth axes and hence to the change in the geographical co-ordinates on the surface of the earth. Here a platform error of 1/10 degree leads to a 6 miles distance error which is independent of time. A change of wander rate of 1/10 degree per hour leads to 6 miles error in an hour's flight, being proportional to time for a constant wander rate.

It is clear that the second system is the only one that is applicable to any lengthy period of operation.

2. Components

The basic components of an inertia system are gyroscopes, accelerometers and integrators.

2.1. GYROSCOPES

There are two basically different types of gyros which are finding application in inertia systems, the integrating gyro and the displacement, or free gyro. In both types, modern development is tending towards obtaining an exact flotation of the inner gyro element in a heavy liquid so that its weight is shifted from the gimbal bearings to the liquid and this greatly reduces the amount of friction at the bearings. Heavy organic liquids such as fluorolubes or mercury have been used and both have advantages and disadvantages: for example a range of fluid densities can be obtained by mixing in the case of the fluorolubes but not in the case of mercury; on the other hand the electrical conducting properties of mercury can be put to advantage.

The integrating gyro is an adaptation of the familiar rate gyro and has only one degree of freedom. In a rate gyro the displacement of the gyro and its gimbal about their free axis is opposed by a spring while in the integrating gyro this is replaced by the viscous damping of the flotation fluid. As a result, the restraining force exerted by the viscous damper is proportional to gyro precessional rate instead of being proportional to precession displacement as in the spring of the orthodox rate gyro. If an integrating gyro is mounted in an aircraft so that its input axis corresponds to the aircraft's yaw axis, for example, a yawing motion causes the gyro to develop a precessional torque which is proportional to the rate of yaw and, because of the viscous damping, this torque results in a gyro gimbal turning rate which is proportional to the yawing rate of the aircraft. The total gyro displacement is therefore directly proportional to the total aircraft displacement in yaw. In order to stabilise a platform, three such gyros with their servo systems are used, one for each of three orthogonal axes of rotation. Any displacement about one axis will, through a pick off from the gyro, generate a signal which, after amplification, is applied to a servo motor to drive the platform in the opposite sense to the aircraft's motion. When three such gyros are used to stabilise an accelerometer platform, then the integrated output signal from an accelerometer, which is proportional to the vehicle's angular velocity relative to the earth's centre, can be introduced into the integrating gyro's torque motor which will cause the gyro to precess and the accelerometer platform to rotate at an angular velocity equal to that which has been sensed by the accelerometer.

This type of gyro is an instrument of high sensitivity, low drift and capable of measuring a wide range of angular rates.

The displacement or free gyro is a gyro with two degrees of freedom and is an extremely refined and accurate version of the conventional gyro which has been used for many years in artificial horizons and gyro compasses. The whole gyro system is mounted in an outer case and any displacement of this case relative to the inner gyro element generates error signals which are fed to servo motors which rotate the stabilised platform.

An advantage of the free gyro is that only two are needed to stabilise a platform instead of the three that are required with the integrating gyro. This is of importance in keeping down the size of the stabilising platform; but the differences between the two types of gyro are more fundamental than this.

The integrating gyro is not really a true stabilising gyro until it is provided with the gimbal servo system of the stabilised platform, this means that the gyro and servo are very intimately linked and any imperfections in servo performance can have serious effects. The free gyro, however, is a stabilising gyro in its own right and is therefore less dependent on the performance of the servo system. Moreover, it is less liable to be damaged by transient movements which, in the case of the integrating gyro, if they are beyond the response of the servos or the restraining force of the viscous damping may appear as large reactions on the gimbal bearings. The de-coupling of the gyro from the platform servo means that vibrations and oscillations of the platform servos are not coupled directly into the gyro element, whereas in the integrating gyro they are coupled through the fluid damping. The heavy viscous damping of the integrating gyro which is essential to it, reduces the stability of the system for high frequency oscillations as compared with the free gyro.

The free gyro has therefore distinct theoretical

advantages, but to realise these advantages requires a great deal of development. The integrating gyro is inherently simple in mechanical design and permits a greater overall stability of the assembly; for this reason it has reached a higher state of development than has yet been reached for the free gyro.

A gyro will maintain the direction of its spin axis so long as it is free from spurious disturbing torques; such torques may arise from a displacement of the centre of gravity when the system is subjected to accelerations (or to changes of orientation relative to gravity), from frictional restraints in gimbal bearings or from disturbances imparted from the surrounding medium. Once the initial balancing has been done it must not be upset by conditions obtaining later such as changes in effective g, temperature changes, vibrations and bumps.

The effect of the spurious disturbing torque is to produce a drift in the direction of the axis which, in a gyro-stabilised platform for example, produces spurious values of acceleration. The aim is, naturally, to keep this drift or wander rate down to a minimum even if one cannot eliminate it completely.

To reduce and unify the effect of acceleration the principles of iso-elastic construction can be adopted. That is to say the construction of the motor, motor housing and gimbal should be so devised that the deflection of the c.g. of the assembly from its integral centre of support is the same for all directions of the applied acceleration.

The advantage of the iso-elastic construction is that in response to any applied acceleration, including gravity, the c.g. will always move from the centre of buoyancy along the line of the applied acceleration and consequently no disturbing torque will be applied to the gyro. If the construction is not iso-elastic the c.g. will not, in general, move along the line of the applied acceleration and a disturbing torque will result which will differ for different directions of application of the acceleration. (For a floating gyro of angular momentum 3×10^6 (c.g. units) the c.g. must not shift more than 10^{-6} in. normal to an applied acceleration of $1g$ if the performance is to be kept below a wander rate of 1/10 degree per hour.)

One way of correcting for lack of iso-elasticity, due to Barnes of the R.A.E, is shown in Fig. 2 which shows an arrangement suitable for a single degree of freedom floating gyro. A compensating weight can be mounted on a leaf spring so that its movement can be increased or decreased to the appropriate value. This adjustment can be made through a suitable gland after the float has been finally assembled in its case. For a two degree of freedom gyro, the leaf spring would be replaced by a round wire parallel to the spin axis.

The performance of gyroscopes will be dealt with later but I should perhaps mention at this point that the criterion of gyroscope performance is its wander rate and the constancy of the wander rate under all operating conditions. It has often been suggested that a steady wander rate might be predictable on the basis of laboratory testing and that allowance could be made

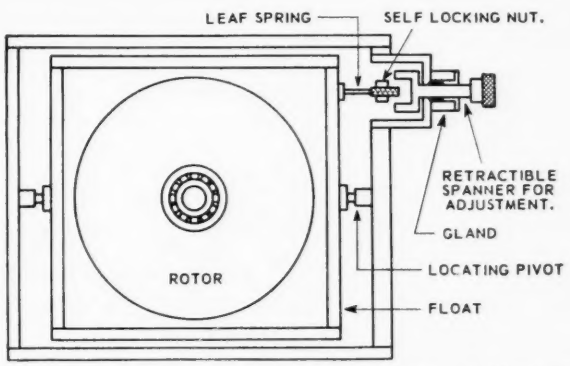

FIGURE 2. A method of compensating a single degree of freedom gyro for non-iso-elasticity.

for it during actual operation. We shall see later how unpredictable it is.

The actual wander rate that can be regarded as acceptable depends on the accuracy required and the time of flight. As an example of the order of accuracy which might be required, one can say that for accurate long range navigation, a wander rate under actual condition of less than 1/20 degree per hour is required which may involve even smaller wander rates under laboratory conditions.

2.2. ACCELEROMETERS

There are many types of accelerometers in general use but for application to inertial navigation systems there are three main types under development, all of which are based on the measurement of the force exerted on a constant mass. The lines of development are the measurement of this force:

(*a*) by balancing against the force exerted by a magnet on an electric current (force feedback system);

(*b*) by balancing against the tension in a wire or crystal, the frequency of vibration of the wire or crystal being used as a measure of the tension;

(*c*) by applying the force as a torque to precess a gyro, the rate of precession being a measure of the acceleration.

The requirements are very stringent since not only has the accelerometer to measure very small accelerations (which means suspending the mass in such a way that forces exerted by the suspension are small compared with the smallest force to be measured) but it also has to be capable of measuring quite large forces in certain applications—such as missiles. At the same time this suspension has to be capable of taking continuous loads at right angles to the direction of measurement of 10^4 to 10^5 times the minimum force; it also has to withstand vibration and gust loads.

The force feedback type seems to have had the most attention devoted to it.

The moving mass consists of a coil moving in the field of a pair of permanent magnets. Any movement of the system is arranged to produce an electric current

in a monitoring system, which might be coil and magnet or a variable capacity in a capacity bridge, which is fed back through an amplifier to the moving coil of the accelerometer to bring the accelerometer mass back to the central position. The force on the accelerometer and, therefore, the acceleration, is proportional to the current, so that the magnitude of the current is a direct measure of the acceleration. There are various difficulties to be overcome in order to produce an instrument with a high order of accuracy. One of these is the production of a suitable permanent magnet, which maintains a constant field strength. The field strength is affected by temperature for which compensation can be applied but it is also affected by mechanical shock and over a long period a steady change takes place. In addition the flow of current through the moving coil has a demagnetising effect on one of the magnets and a magnetisation effect on the other, which leads to inconstancy and specifically affects the linearity of the system.

An important general factor is the necessity to ensure that the axis of the instrument is in line with its sensitive axis. Normally the axis is perpendicular to an accurately ground mounting face and the aim is to ensure that the sensitive axis is less than one minute of arc away from the physical axis determined by the perpendicular to the mounting face.

We usually specify the performance of an accelerometer by three parameters, namely zero error, scale accuracy and cross-coupling. The first two are self-explanatory but a word on cross-coupling is perhaps necessary. If the accelerometer "mass" moves relative to the "frame" in a straight line in the presence of cross accelerations, then these cross accelerations have no effect on the output achieved in the active direction. This is seldom realised in practice, many accelerometers in fact have the mass moving along a small arc of a circle, and the effect of the departure from the straight line is to introduce an output error which is related to the product of the cross and active acceleration components. In general, to reduce the cross-coupling means having an accelerometer with small linear displacement and therefore a high natural frequency.

2.3. INTEGRATORS

We shall assume first that accelerometers and integrators are separate devices and we can consider their accuracies separately. There are two basic types of integrators in use; the electronic such as the Miller type and the tachogenerator type. Both of these can give high accuracies if due care is taken with the components and the important factors are:

(*a*) maximum input range,
(*b*) zero uncertainty,
(*c*) scale error,

all of which are self-explanatory.

Another important feature, however, is the speed of response. If the integrator is of the type using a tachogenerator the initial lag in the components could lead to unacceptable errors. The first integrator stage is the one in which this possible source of error should be particularly watched and, in missile applications, where the accelerations may be large and of short duration, this effect will assume great importance. To obviate this source of error we tend to favour electronic integrators, especially in the first stage.

It is generally agreed that separate accelerators and integrators, which are connected electrically, will give the most flexible scheme and will lead to the smallest accelerometer unit thus saving valuable platform space and it also allows us to take off the accelerometer and first integrator outputs, as well as the second integrator output, which is necessary in mixed systems, for example. Nevertheless, there are possible applications where it is convenient to have a coupled accelerometer and integrator (or integrators). One interesting possibility leads from the use of a gyro as an accelerometer which I mentioned before. Such a gyro has been developed with an out of balance mass in line with the spin axis. The gyro is mounted with float and spin axis normal to the direction in which the acceleration is to be measured (which is parallel to the platform axis). A pick-off signal from the gyro acts through a servo to maintain the spin axis normal to the platform axis. The total revolutions of the platform due to gyro precession are proportional to the first integral of acceleration acting on the out of balance weight. This is potentially a very accurate integrator if the gyro wheel speed is accurately controlled. This type was first used in the German V.2 rocket as a velocity meter.

2.4. SERVOS

An important part is played by the servo-systems which are required to provide the accurate stabilisation of the accelerometer platforms. Obviously the development of the servo-system must keep pace with the development of gyros and accelerometers, otherwise the whole of that development will be wasted. However, in general, there are no new problems introduced and standard servo techniques and theories are applicable to inertial equipment.

3. Testing of Gyroscopes

3.1. PURPOSE OF GYRO TESTS

Tests of gyroscopes as components, as distinct from parts of systems, may be sub-divided as follows:

(*a*) Production acceptance tests to ensure satisfactory performance and freedom from known faults.
(*b*) Calibration tests—these are generally required so that the best use may be made of a gyroscope in the system.
(*c*) Development, evaluation and assessment tests. These are generally aimed at determining how the gyro performance varies with time, environmental conditions, and so on. In this work, the aim should be to devise tests which will separate the effects reliably so that they may be studied individually, and use the ones which will be described.

3.2. QUANTITIES TO BE MEASURED

With the introduction of flotation gyros, in which a fluid support is used to take practically the whole of the

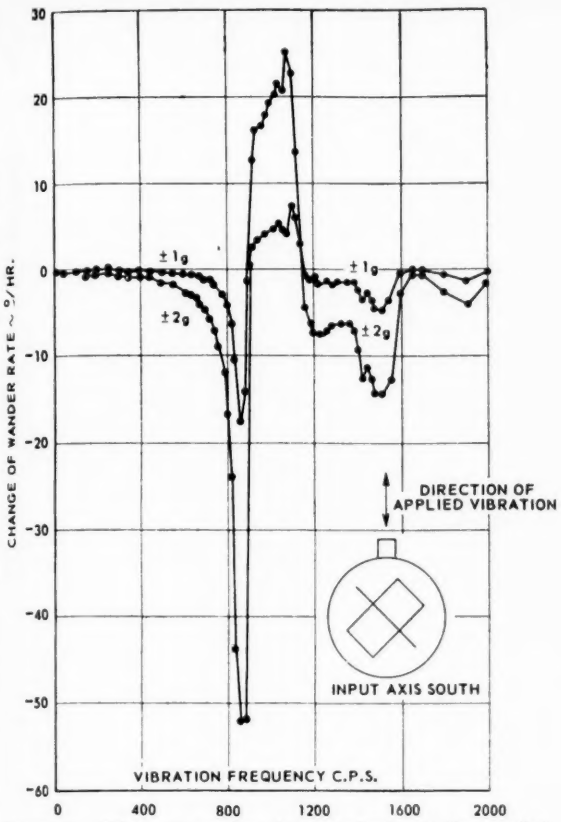

FIGURE 3. Effect of varying the magnitude of the applied vibration at 45°.

load off the gimbal bearings, the magnitude of the gimbal friction is much reduced and it has ceased to be the predominant source of gyro drift or wander. At the same time this fluid support is rather more definite in its line of application than a ball bearing in which neither the balls nor the tracks are perfectly round, nor are the balls the same diameter to the required degree of accuracy (considerably less than one wavelength of light). Nothing has been done, however, which would affect the stability of the c.g. either for better or for worse. The net result is that the random wander is reduced and systematic effects are noticeable, especially within any one warm-up—at a very much lower level than heretofore—and people began to study and attempt to compensate for these "systematic effects."

According to elementary theory, gyro wander can be broken down into wander caused by constant torques, wander by torques proportional to acceleration, and wander caused by effects proportional to the square of the acceleration such as an-isoelasticity (*loc cit*).

3.3. FACTORS CONTRIBUTING TO GYRO WANDER

The factors contributing to gyro wander normally considered are:

1. *Non-Acceleration-Sensitive Torques.* These comprise the parasitic torques due to the ligaments (or flex leads) used to feed current to the spin motor, the magnetic reaction of the take-off and the parasitic reaction of the torque motor. All are normally considered as behaving like linear springs with a certain amount of initial deflection at the gimbal null. The ideal torque motor should of course exert a torque which is unvarying with position and which passes accurately through zero at zero control current, but most real torque motors have their failings.

2. *Acceleration-Sensitive Torques.* The acceleration sensitive torques are treated as being mass unbalances. In the case of exact flotation there is no load on the gimbal pivots and a torque can only arise as a result of the centre of gravity and centre of buoyancy not being coincident, but in the case of imperfect flotation (*i.e.* the float either under-buoyant or over-buoyant) they can also rise as a result of the gimbal pivot axis not passing through the centre of gravity and centre of buoyancy. In the case of a single degree of freedom gyro, mass unbalance is only critical around the output (gimbal) axis and it is generally resolved along the spin axis and along the input axis, a procedure that can often mask the true magnitude of the vector sum.

In the case of the two degree of freedom gyro, mass unbalance has to be evaluated:

(*i*) Along the spin axis and perpendicular to both the spin axis and the inner gimbal pivot axis for the float alone.

(*ii*) Along the spin axis and along the inner pivot axis for the float plus inner gimbal.

3. *Acceleration-Sensitive Torques.* Although an-isoelastic torques are the direct effect of masses being mounted on elastic supports, they are normally treated as being d.c. effects and arrangements made to minimise them on this basis. Because, however, the effect is proportional to the square of the acceleration, the sign of the output (wander) is independent of the sign of the applied acceleration, but it does depend on the direction. As a matter of convenience therefore an-isoelasticity is generally measured under vibration conditions, although it has been measured by observing the performance of the gyro in various attitudes in a 1*g* field. Recent work at R.A.E. and elsewhere has, however, shown that if the gyro is vibrated at the natural frequency of the mass on its electric support, then the an-isoelastic wander rate can become enormous (*see* Fig. 3).

The non-acceleration sensitive torques may be approximately evaluated by testing the gyro with its output (float) axis vertical (so that the mass unbalance can exert no couple around the output axis) and then correcting for the component of earth's rate applied to the input axis, or, alternatively, by making one test, then rotating the gyro through exactly 180 degrees around the vertical output axis and making a second test with the component of earth's rotation reversed.

Another method is to mount the gyro with its output axis vertical and observe the orientation of the input axis at which the non-acceleration sensitive torques produce a precession rate exactly equal to the horizontal component of earth's rate around the input axis.

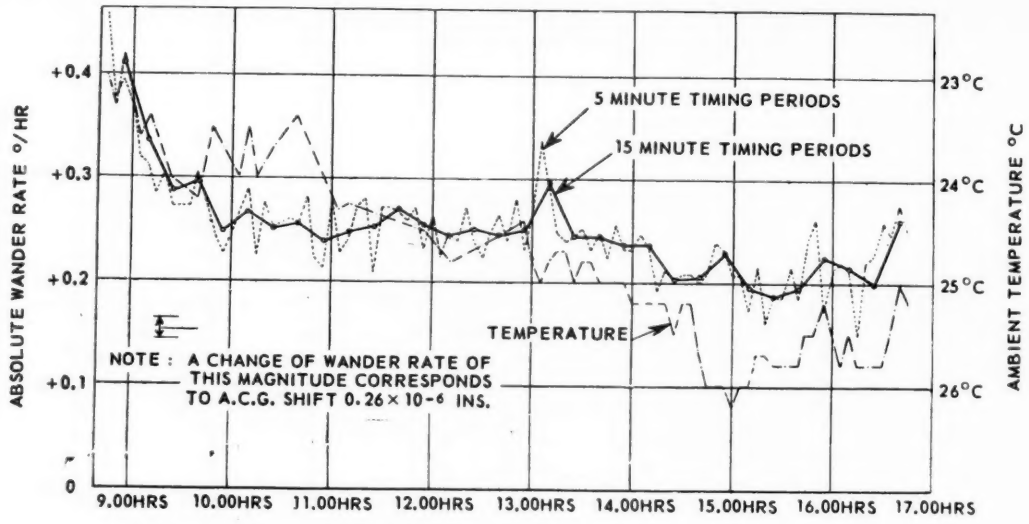

FIGURE 4. Variation of wander rate with time.

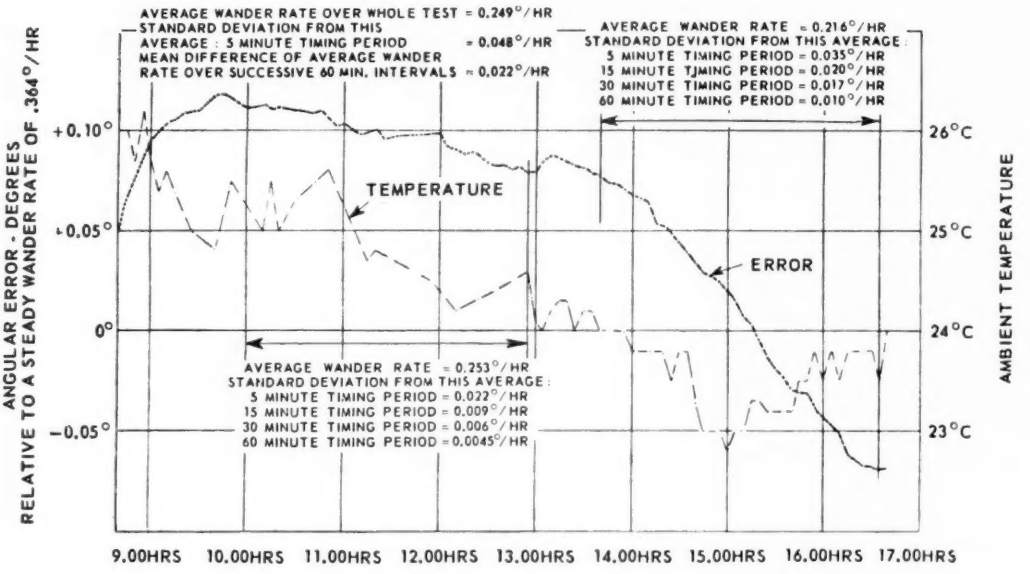
AVERAGE WANDER RATE OVER WHOLE TEST = 0.249°/HR
STANDARD DEVIATION FROM THIS
AVERAGE : 5 MINUTE TIMING PERIOD = 0.048°/HR
MEAN DIFFERENCE OF AVERAGE WANDER
RATE OVER SUCCESSIVE 60 MIN. INTERVALS = 0.022°/HR
AVERAGE WANDER RATE = 0.216°/HR
STANDARD DEVIATION FROM THIS AVERAGE :
5 MINUTE TIMING PERIOD = 0.035°/HR
15 MINUTE TIMING PERIOD = 0.020°/HR
30 MINUTE TIMING PERIOD = 0.017°/HR
60 MINUTE TIMING PERIOD = 0.010°/HR
ANGULAR ERROR - DEGREES RELATIVE TO A STEADY WANDER RATE OF .364°/HR
+0.10°
+0.05°
0°
−0.05°
TEMPERATURE
ERROR
AVERAGE WANDER RATE = 0.253°/HR
STANDARD DEVIATION FROM THIS AVERAGE :
5 MINUTE TIMING PERIOD = 0.022°/HR
15 MINUTE TIMING PERIOD = 0.009°/HR
30 MINUTE TIMING PERIOD = 0.006°/HR
60 MINUTE TIMING PERIOD = 0.0045°/HR
26°C
25°C
24°C
23°C
AMBIENT TEMPERATURE
9.00HRS 10.00HRS 11.00HRS 12.00HRS 13.00HRS 14.00HRS 15.00HRS 16.00HRS 17.00HRS

FIGURE 5. Error of reference direction with time—Test A.

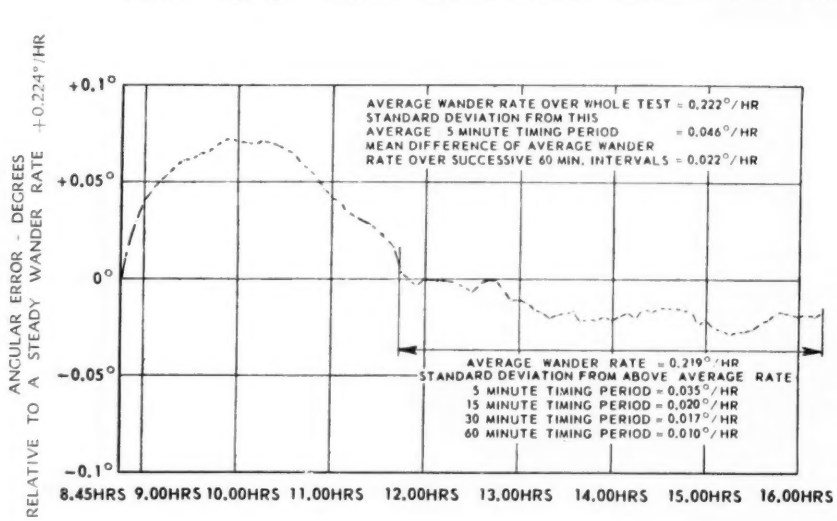
ANGULAR ERROR - DEGREES RELATIVE TO A STEADY WANDER RATE +0.224°/HR
+0.1°
+0.05°
0°
−0.05°
−0.1°
AVERAGE WANDER RATE OVER WHOLE TEST = 0.222°/HR
STANDARD DEVIATION FROM THIS
AVERAGE 5 MINUTE TIMING PERIOD = 0.046°/HR
MEAN DIFFERENCE OF AVERAGE WANDER
RATE OVER SUCCESSIVE 60 MIN. INTERVALS = 0.022°/HR
AVERAGE WANDER RATE = 0.219°/HR
STANDARD DEVIATION FROM ABOVE AVERAGE RATE
5 MINUTE TIMING PERIOD = 0.035°/HR
15 MINUTE TIMING PERIOD = 0.020°/HR
30 MINUTE TIMING PERIOD = 0.017°/HR
60 MINUTE TIMING PERIOD = 0.010°/HR
8.45HRS 9.00HRS 10.00HRS 11.00HRS 12.00HRS 13.00HRS 14.00HRS 15.00HRS 16.00HRS

FIGURE 6. Error of reference direction with time—Test B.

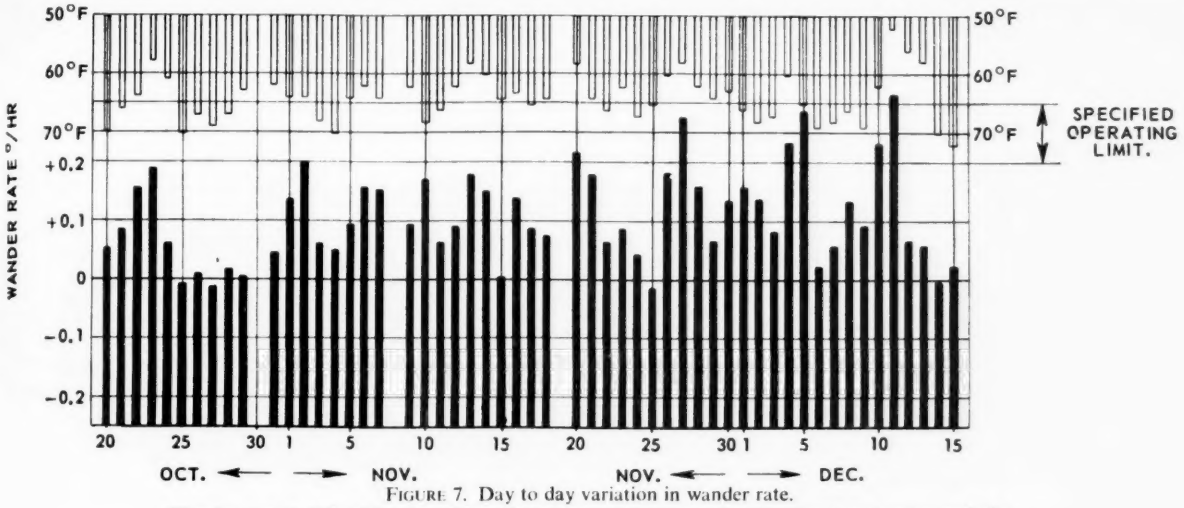

FIGURE 7. Day to day variation in wander rate.
Wander rate is defined by the arithmatic average of 5 consecutive 15 minute measuring periods.
Gyro continuously maintained at constant temperature by thermostatic control accurate to $\pm\frac{1}{2}$°F.
Tests commenced 5 minutes after powering rotor (synchronous in approx. 4 minutes).
Standard deviation from mean 0·084 °/hr. R.M.S. value of day to day variation = 0·065 °/hr.

Yet another method is to test the gyro in succession in six mutually perpendicular positions with the gravity vector along each of the principal axes in turn. Then the gravity sensitive and non-gravity sensitive terms can be evaluated mathematically from the changes in precession rate observed when the direction of the g vector is turned to 180 degrees relative to the gyro.

According to simple theory, the quantities described are sensibly constant and represent the major part of the wander rate. When they are removed by a compensating torque applied by means of the torque motor, only the random wander is left. In the course of one run under *completely undisturbed conditions*, this is generally so (after the first hour—for samples of the apparent performance that can be achieved) especially when a suitable means of analysis is chosen (*see* Figs. 4, 5, 6).

Very little experience indeed, however, is required to learn that any disturbance at all generally produces changes in the co called steady components of wander rate which are large compared with random wander rate. Figs. 7 to 10 show the effects of various types of disturbances on the so called steady wander rate.

3.4 EFFECT OF CHANGE OF TEMPERATURE

In an attempt to determine the effect of change of operating temperature, the setting of the gimbal thermostat was altered on several gyros and the wander rate

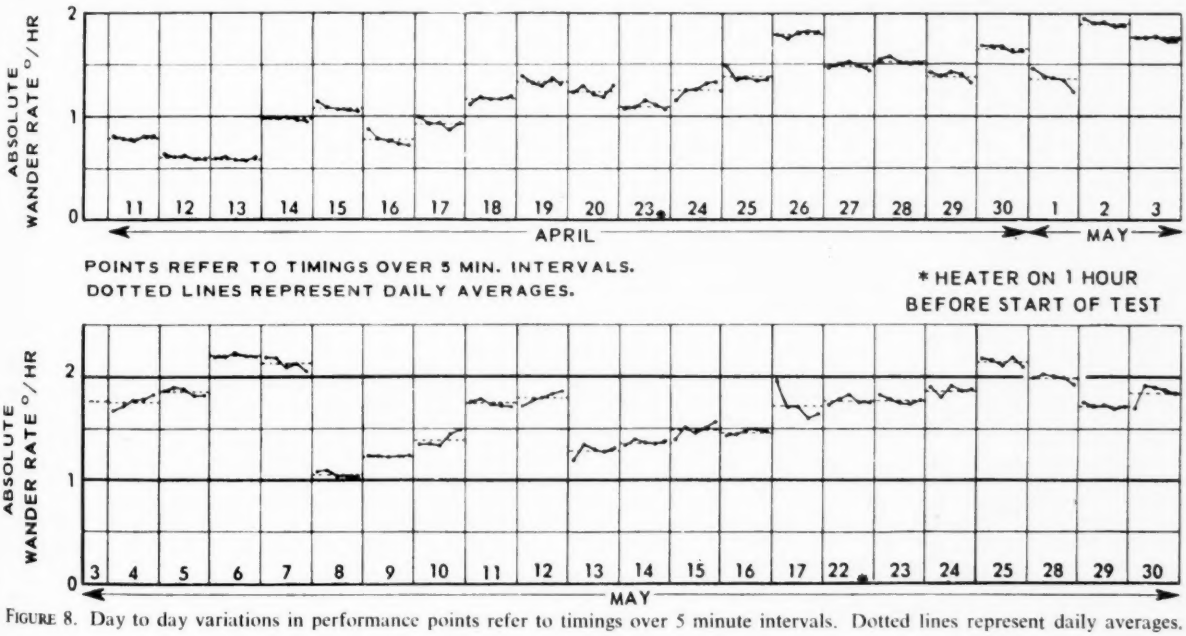
ABSOLUTE WANDER RATE °/HR
2
1
0
11 12 13 14 15 16 17 18 19 20 23* 24 25 26 27 28 29 30 1 2 3
APRIL
MAY
POINTS REFER TO TIMINGS OVER 5 MIN. INTERVALS.
DOTTED LINES REPRESENT DAILY AVERAGES.
* HEATER ON 1 HOUR BEFORE START OF TEST
ABSOLUTE WANDER RATE °/HR
2
1
0
3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 22* 23 24 25 28 29 30
MAY

FIGURE 8. Day to day variations in performance points refer to timings over 5 minute intervals. Dotted lines represent daily averages.

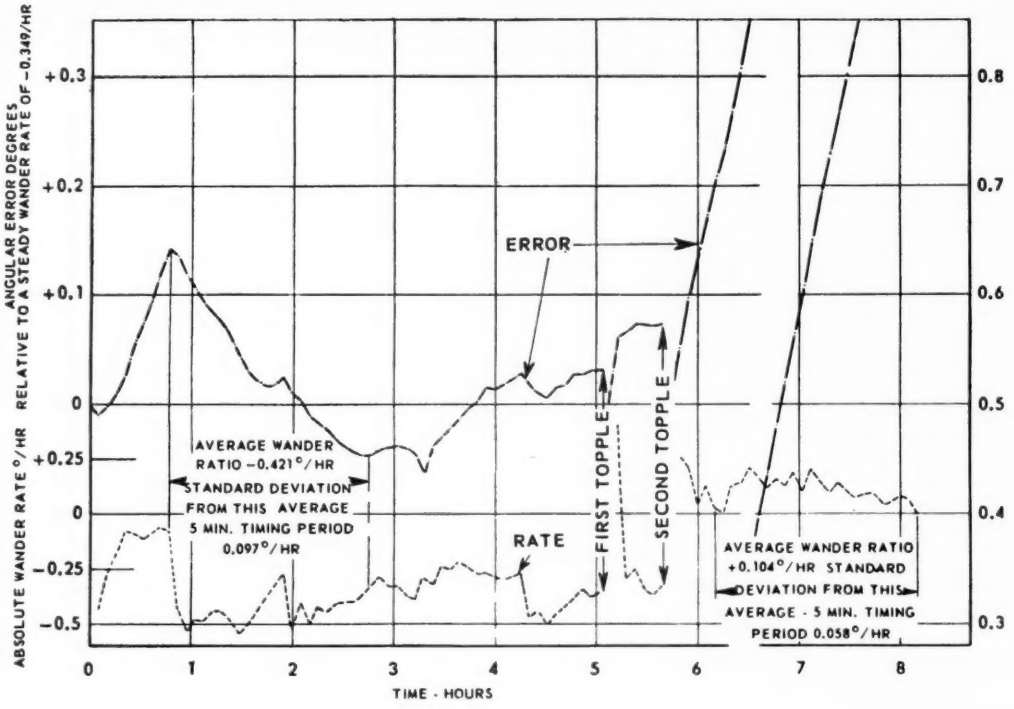

FIGURE 9. The effect of toppling the gyro.

measured at a variety of temperatures, but otherwise under identical conditions. In the case of good gyros it was found that the wander rate/temperature curve plotted as a smooth line, but in the case of inferior gyros there was considerable scatter. The mechanism of the change of wander rate is believed to be partly thermal distortion of the float and partly change of the relative densities of the float and flotation fluid, coupled with the gimbal axis not passing through the centre of buoyancy. The cause of the scatter of the points may have been either friction on the gimbal pivots or sudden slipping of joints under the influence of thermal stresses within the gyro.

3.5. SPURIOUS WANDER

In general the only possible source of information regarding the rotation of the float and hence the performance of the gyro, comes from the electrical take-off and any spurious electrical signals in its output will accordingly lead to a false assumption of gyro wander being made.

3.6. DESIGN FAULTS

In addition to design imperfections, such as lack of isoelasticity or undue sensitivity to spin motor supply voltage, certain gyros suffer from faults such as the wander rate changing with changes in external magnetic

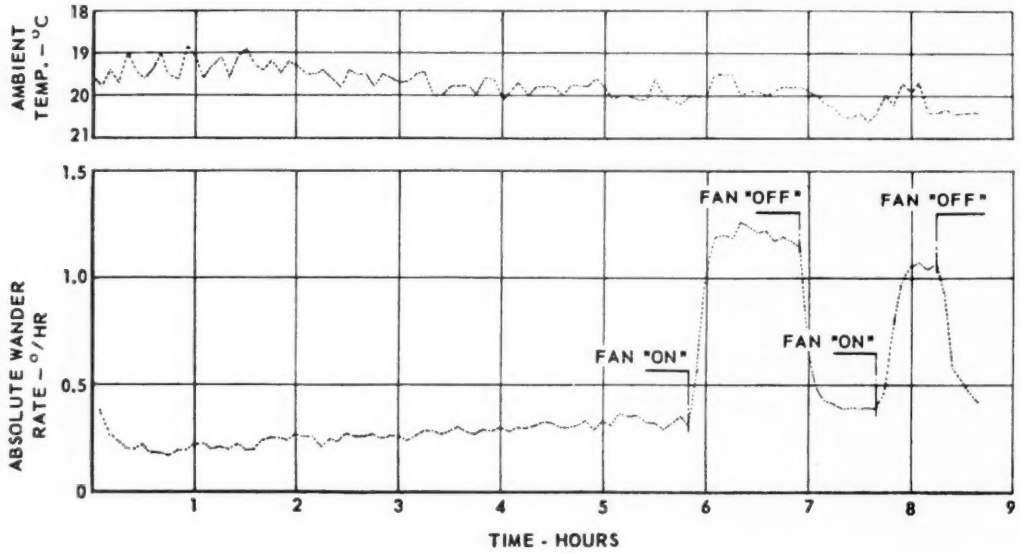

FIGURE 10. The effect of a draught.

fields or changes in the atmospheric pressure. Sensitivity to external magnetic fields generally comes from the use of moving magnet torque motors, together with non-existent or inadequate magnetic screening. Such a fault is most embarrassing, since the wander rate varies with heading or with the presence or absence of magnetic material or stray magnetic fields, such as those caused by the electrical wiring in an aeroplane.

Sensitivity to changes in atmospheric pressure may arise as a result of the expansion bellows being open to the atmosphere on one side, with the result that changes in atmospheric pressure are transmitted to the float which is thereby distorted. They can be overcome by sealing the outside of the expansion bellows in an evacuated chamber and making use of a spring to maintain the requisite hydrostatic pressure. If the chamber is not evacuated, then changes in ambient temperature will cause expansion or contraction of the residual gas and hence a change in pressure.

3.7. MANUFACTURING FAULTS

One of the faults which gives much trouble during manufacturer's test, and to some extent later, is sticking of the float. Sticking is perhaps a misnomer since a torque of a few dyne-centimetres is generally enough to free it, but in precision gyros such a parasitic torque is catastrophic. One of the difficulties is that sticking often occurs in certain specific attitudes only and possibly only at certain specific angular positions of the float away from the null. This makes testing for, and location of, the fault exceedingly tedious. The exact causes of float sticking are generally unknown, but the following are suspected:

(*a*) Dust and/or hairs in the flotation liquid.
(*b*) Fouling in the float gap (approximately 0·005 in. on single degree of freedom gyro) in the torque motor air gap or in the pick-off air gap.
(*c*) Damaged gimbal pivot bearings.
(*d*) A flex lead deflecting under gravity or buoyancy forces and touching in certain attitudes.
(*e*) Corrosion of metal parts.
(*f*) Gumming, sludging, segregation or stratification of the flotation liquid (fluorolubes are generally mixtures and will segregate with time).
(*g*) Distortion of metal parts subsequent to machining or swelling of the epoxy resins.
(*h*) The presence of small air bubbles, particularly in the float, torque motor or take-off air gap.

The usual remedial action involves dismantling the gyro, the sequence being to drain it, examine all parts, clean all parts, refill and retest.

Preventative action is generally centred around deburring and cleaning operations. All parts, however small, are thoroughly deburred, often with dental tools and all sharp edges are removed. Ultrasonic cleaning in several stages is then employed, followed by assembly in the cleanest area of the two or three stage clean assembly area. It is hard to find superlatives adequate to describe the extent of the care necessary—probably the best way of describing it is to say that the standard of cleanliness aimed at is to that of a watch-maker as a watch-maker's is to that of the local garage.

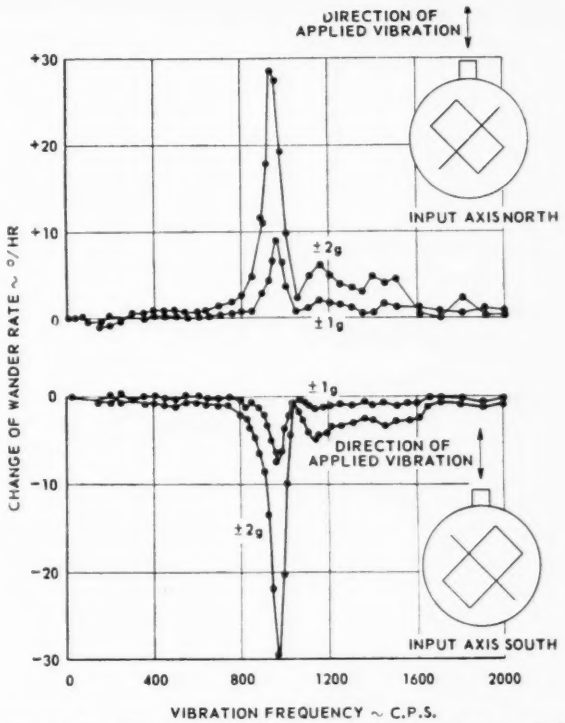

FIGURE 11. Effect of varying the magnitude of the applied vibration at 45°.

The presence of small air bubbles in the flotation liquid has been mentioned already; another possible production fault is the presence of medium or large air bubbles. The effect of such air bubbles is that when the direction of the acceleration vector is changed suddenly, *e.g.* by rotating the gyro about a horizontal axis in the laboratory, the float appears to have acquired additional unbalance until such time as the air bubble can find its way to the top of the liquid space. The transient effect may last between a few seconds and, say, 15-30 mins.

The presence of air bubbles generally arises from imperfect filling, but it can arise as a result of the evolution of gas from imperfectly cured epoxy resins. It is quite natural that production supervisors are loth to allow the necessary time for de-gassification of parts and the achievement of the necessary high vacuum before filling, but time spent on these operations can be very well worthwhile in terms of saving of trouble later.

Another apparently similar effect can be caused by the presence of viscous liquid within the float, caused by leakage. This effect can be expected to increase with time.

Inadequate or lost rotor bearing pre-load generally manifests itself as uncertainty in the c.g. position of the float, but in bad cases it can manifest itself as vibration and/or nutation of a two degrees of freedom gyro. Manufacturers differ in the method which they use to measure bearing pre-load, but one of the most common is to measure the rate of rundown of the rotor after the

spin motor power has been shut off. Since at high speeds, windage is predominant, it is usual to employ electronic detection apparatus to sense the rotation of the hysteresis motor and to time the deceleration over the last few revolutions of the wheel—say below 20 r.p.m. Another method that has been proposed is to take advantage of the fact that the stiffness of the bearing increases with the pre-load and to measure the natural frequency of the rotor on its bearings. So far all known attempts have ended in failure owing to the difficulty of exciting and measuring the resonant frequency. It is believed that if the test were done on a completed gyro and the method combined with the measurement of the an-isoelastic properties under vibration conditions, then the method would most likely be successful—*see* Section 3.2 and Figs. 3 and 11, which show the sharpness of the resonant peaks. The need to perform the test on a completed gyro would, however, preclude it being used in the early stages of manufacture.

Another manufacturing fault experienced has been undue instability of the position of the c.g. Whether this is due to bad workmanship, bad fits at the joint faces, bad potting of the electrical components, and so on, is not known—it probably varies from gyro to gyro—but since a movement of the c.g. of 10 micro-inches (half a wavelength of yellow light) will generally produce a change in wander rate of more than one degree per hour in a 1*g* field, it is very hard to know what to do about it. In this connection, it is instructive to remember that if the total weight of the float of a given gyro is 500 grams, the weight of the cage in each of the rotor bearings is 0·25 gram, the total free movement of the cage is 0·001 in., then the total movement of the c.g. of the float due to both cages moving together is one micro-inch.

Yet another gyro fault is gimbal pivot friction but whether all cases of it can be classed as manufacturing faults is open to question. In some cases actual faults have been found in the gimbal pivot bearings and the existence of friction is indisputable, but in other cases all that can be said is that to describe the effect observed from outside the gyro as friction provides the most realistic and complete picture of its characteristics.

A case in point is shown in Fig. 12. This is a copy of part of the chart record obtained during a "closed loop" wander rate test on an azimuth gyro mounted on an optical goniometer (for details of the closed loop method *see* later). Reading the chart from left to right it shows that $2\frac{1}{4}$ hours after switch on—after full thermal stability had been reached—the turntable and gyro were turned through the very small angle of 11 secs. of arc from north to east. Since the chart record is a record of precession rate this naturally appears as a change of rate, but, contrary to expectation, the wander rate does not return to its previous value within a time consistent with the time constant of the servo, but only to within about one degree an hour of its previous value and thereafter takes approximately 15 mins. to return. Thirty minutes later when a further rotation of 14 secs. of arc was made, the wander rate never did return exactly to the previous value. When the rotation of the table was increased to 60 secs. of arc the value of the permanent change in wander rate was also increased, although some reduction of the change was made about an hour later, when the gyro was thoroughly exercised by flexing the supports of the turntable through about ± 2 secs. of arc. This effect has been found to be present to a varying degree in a great many gyros although certain samples are completely free from it. The real cause is not known but the effect seems to be exactly as if rolling movement takes place between the gimbal pivot and its jewel for small angular movements, and slipping takes place for movements in excess of say 10 secs. of arc—after the angle of friction has been reached—but attempts to prove or disprove this by changing the load on the gimbal pivots by changing the operating temperature of the gyro and hence the unbuoyancy of the float have met with no success. The fact that the effect is absent on certain gyros means that it is not a fundamental defect.

Another likely gyro fault lies in the proportioning of the rotor bearings and the adherence to these proportions in production. In the event of the rotating member of the bearing being round but eccentric, the resulting error can be eliminated in the dynamic balancing of the rotor, but in the event of one or both bearing tracks being out of round, and/or the balls being out of round and/or of different diameters, then the centre of rotation of each bearing will be subjected to a periodic motion dependent on the epicyclic gear ratio of the bearing and the contours of the components. These periodic motions of the bearings will result in tilts of the spin axis of the rotor with respect to the gimbals and take-offs—1/5th sec. of arc tilt per micro inch movement at one inch bearing spacing—and in the event of their frequency coinciding with the nutational frequency of the gyro or the natural frequency of the servos, then serious trouble will result. Considerable care is therefore required in the proportioning of the rotor bearings to

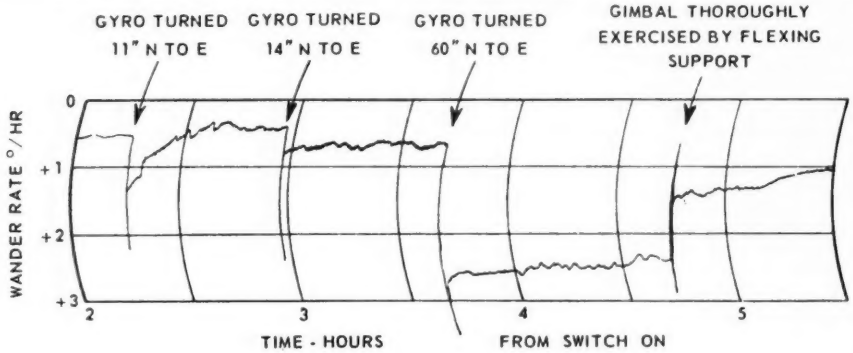

FIGURE 12. Apparent sticking of float. Measurements made by recording torque motor currents.

ensure that exact repetition of the relative positions of the balls and tracks only occurs at infrequent intervals and at frequencies well away from the natural frequencies of the system.

3.8. METHODS OF TEST

This section is primarily concerned with methods of measuring wander rate—*i.e.* the effects of mass unbalance and other parasitic torques—and only secondarily with the calibration of torque motors take-offs and the like. Precision gyros for inertia navigation are primarily required for the angular stabilisation of platforms and the important quantity is therefore not the instantaneous value of the wander rate but the variation of the error angle with time.

There are three main methods of measuring gyro wander rate as follows:

3.8.1. *Single Axis Platform Tests*

In these tests the gyro is used to stabilise a single axis servo controlled table whose motion relative to the earth is measured; then, providing that the attitude of the gyro relative to the axis of rotation of the earth is known, the correction for the relevant component of earth's rotation rate can be made and the steady wander rate of the gyro and the variations therefrom can be evaluated. This test can be performed in a variety of attitudes and measurement is generally made by timing the rotation of the table through successive equal angular intervals of ten minutes, 1, 3 or 5 degrees. A line diagram of a single axis stabilised platform, with an additional take-off for angular measurement, is given in Fig. 13.

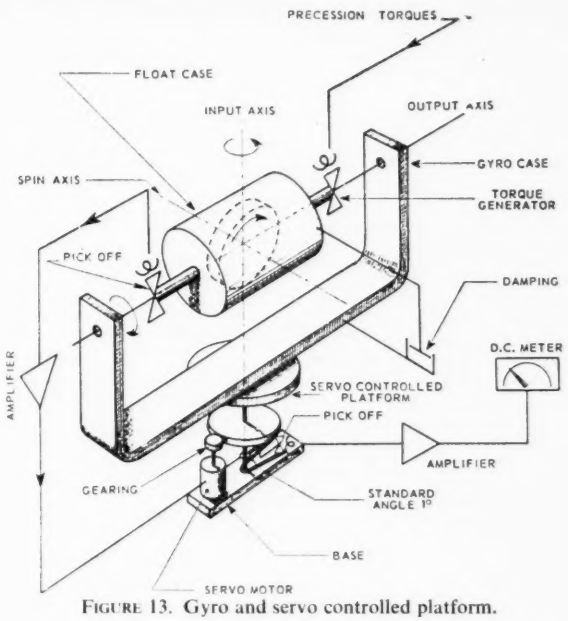

FIGURE 13. Gyro and servo controlled platform.

3.8.2. *Closed Loop Servo Tests*

In these tests the output from the gyro take-off is taken through an amplifier to a phase sensitive rectifier and back to the gimbal torque motor, so that the sensitive element is servo constrained to follow movements of the case. Under these conditions the gyro behaves as a rate gyro with an "electrical spring" and the current through the torque motor provides an instantaneous indication of the precession rate demanded by the take-off. If the gyro is rigidly clamped to ground, the rotating earth can be used as a precision turntable and after subtracting the relative component of earth's rotation the wander rate can be evaluated in any attitude.

Superficially the fact that this test gives a measure of instantaneous wander rate appears to be an advantage, but experience shows that very great skill is required in interpreting the records obtained, since in order to obtain accurate results the servo time constant has to be of the order of a few seconds and for a time constant of t seconds, only one second angular deflection of the take-off is required to call up a precession rate of $1/t$ degrees per hour. This extreme sensitivity to small angular movements of the take-off means that wholly irrelevant angular inputs of a few seconds of

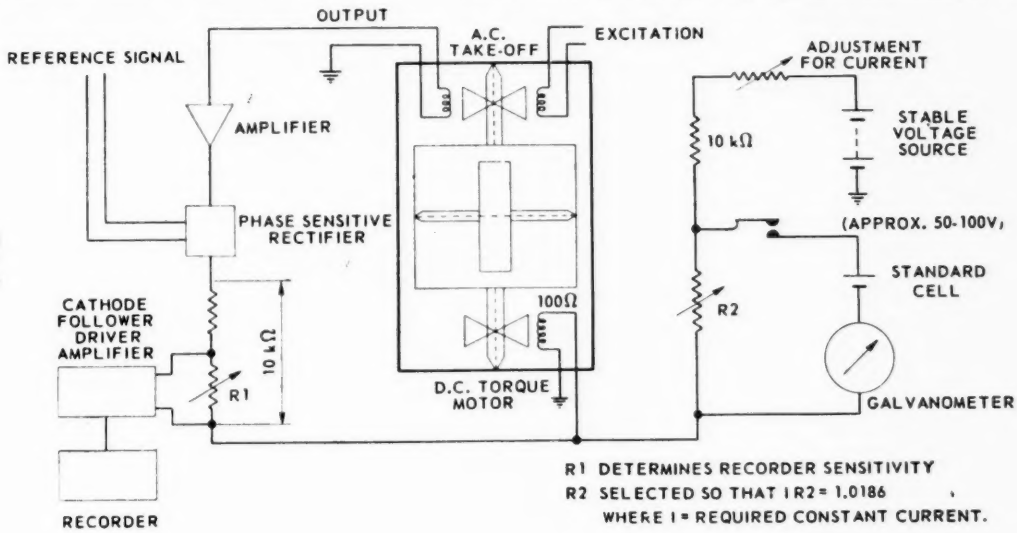

FIGURE 14. Closed loop gyro test circuit (with zero suppression.

arc can cause a large apparent variation in wander rate since one second change of heading in 0·5 sec. of time corresponds to a rate of 2 degrees per hour. Fig. 14 shows a line diagram of the electrical connections for performing the closed loop gyro test, with electrical zero suppression in order to obtain a large scale chart record without loss of accuracy. In this circuit the value of the constant current I is monitored by comparing the IR drop across a known resistance R with standard cell, and the source impedances of both the constant current and amplifier circuits are made high, so that the currents can be added in the gyro torque motor without interaction.

3.8.3. *Open Loop Tests*

In these tests the output from gimbal take-off is taken to sensitive electrical measuring gear and the movement of the sensitive element relative to the case is thereby measured, with the gyro subjected to torque motor currents, or components of earth's rotation, or both. If the test is performed by measuring the torque motor current required to hold the sensitive element stationary at or near the gimbal null—*i.e.* exactly to neutralise the wander rate plus the component of earth's rotation—then the test can be applied to both position and single axis rate integrating gyros, but if it is desired to measure the rate of precession of the sensitive element, the test can only be applied to rate integrating gyros: in general it is qualitative only, since in some attitudes the measurement of wander rate is bound to be made in the presence of a component of earth's rate considerably greater, and a temperature change of only 6-8° centigrade is required to double or halve the viscosity of the fluorolube and hence to double or halve the gyro transfer function. Nevertheless, if use is made of a recorder to indicate the smoothness of the precession of the sensitive element, a test can be a valuable one for assessing quickly the freedom of the gyro from friction in a variety of attitudes, both away from and at the gimbal null.

Although the single axis stabilised platform test can be applied in any attitude, it has its greatest value in two positions:

(*a*) As an azimuth platform with the platform axis and gyro input axis vertical. In this attitude the mass unbalance contribution to the gyro wander rate is constant since the direction of gravity does not change with platform precession: also the vertical component of earth's rate in any given latitude is unchanged by azimuth heading. If the platform axis or the input axis of the gyro are not truly vertical, then a sinusoidal component of the horizontal component of earth's rotation rate will appear in the wander rate according to heading and if the gyro is affected by stray magnetic fields, then the effect of earth's magnetic field will vary with attitude.
This is the condition under which many of the R.A.E. tests are done.

(*b*) With the platform axis and gyro input axis parallel to earth's axis so that the platform is subjected to the full earth's rate. Under these conditions the gyro remains stationary in space and the earth's gravity vector rotates around it in a cone once per *siderial* day.

In yet another variation of this test the attitude of the platform is set as in (*a*) for azimuth stabilisation, and an unvarying current of such a magnitude as exactly to oppose earth's rate plus gyro wander rate is then passed through the torque motor so that the gyro remains stationary relative to the earth. Continuous monitoring of the error angle is then possible by optical methods, such as autocollimation, without the need to measure large angles. The disadvantage of this method of test is that a completely seized gyro, a stationary rotor or a break in the take-off or servo circuit can give a result which looks like zero wander.

3.8.4. *Vibration Tests*

In the vibration tests done at R.A.E. a moving coil linear vibrator like a giant moving coil loudspeaker mechanism has been used, power being derived from an audio frequency oscillator and one kW. power amplifier. The type of vibrator chosen was one with its moving coil supported on two well spaced centering spiders so as to give the maximum angular rigidity. The vibrator was mounted with its axis vertical and the gyro was attached on a special bracket designed to combine the maximum of rigidity with the minimum weight, and the centre of gravity of the whole accurately on the thrust line of the vibrator. To check that the vibratory movement of the gyro was purely linear and free from tilts, plane mirrors were mounted on the gyro and an autocollimator was used to check for absence of angular movement around three mutually perpendicular axes throughout the frequency range. In order to avoid unwanted dynamic amplification due to resonances and to get a good measure of the acceleration applied to the gyro, a crystal (G.E.C. type E barium titanate accelerometer) vibration pick-up, feeding into a cathode follower and valve voltmeter, was used. This vibration pick-up was known to have a sensibly level response above about 20 c./sec. and it was calibrated around 40 c./sec. by optical means.

The vibration tests included a continuous search throughout the frequency range 20-2,000 c./sec. with the acceleration maintained at its specified value, as measured by the crystal pick-up and valve voltmeter, and spot readings were taken by (*a*) reducing the vibration to zero, measuring the torque motor current, (*b*) increasing the vibration to the specified level noting the difference in torque motor current, and (*c*) again reducing the vibration to zero and measuring the torque motor current. In the event of the torque motor current not returning to its original value, the test was repeated and the matter investigated further if necessary.

3.9. POLAR AXIS TUMBLING TEST

A line diagram of the apparatus used at the R.A.E. for the polar axis tumbling test is shown in Fig. 15. The test is only applicable to one degree of freedom gyros.

The equipment consists of a rigid spindle inclined upwards so as to point accurately at the celestial pole.

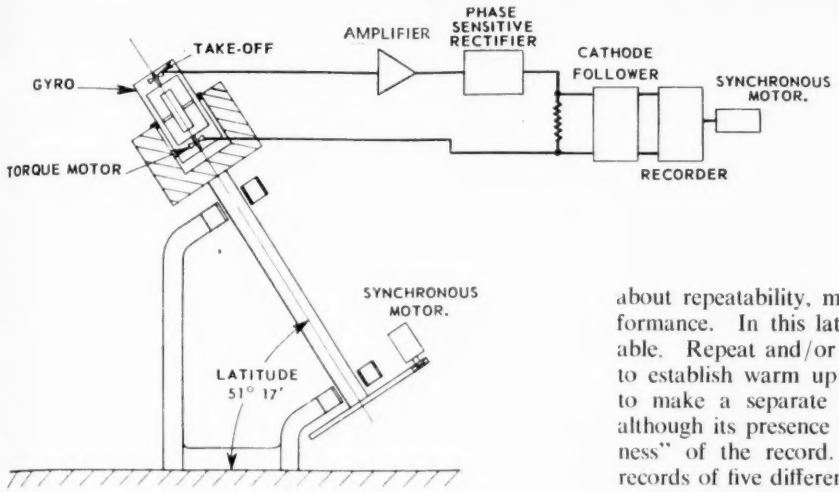

FIGURE 15. Polar axis test rig.

At its upper end it is provided with a bracket to mount the gyro with its output axis parallel to the spindle axis and either the torque motor end upwards, or the take-off end upwards, at will. At the lower end there is a synchronous motor and gearbox to provide the drive. On the present model there is only one speed—130° per hour in either direction—and the use of flying leads limits the test to 3 or 4 complete revolutions in either direction. The torque motor current is measured on a chart recorder with centre zero: the chart recorder being driven by a second synchronous motor operated from the same supply as the first. In a new test rig at present under construction, slip rings are used to permit of continuous running and there are three alternative speeds namely earth's rate (15° per hour), 10 times earth's rate (150° per hour) and 50 times earth's rate (750° per hour).

Provided that the gyro is accurately made, rotation about the output axis has no component around the input axis—it only loads the gimbal pivots against their jewels. In the case of a gyro of $I\omega = 6{\cdot}25 \times 10^6$ being rotated at 130° per hour "with the sun" (115° per hour = 2 radians per hour relative to space), with gimbal pivots one millimeter diameter spaced 10 centimeters apart and having a coefficient of friction steel-jewel of 0·1, the precessional load of each pivot is approximately 0·35 grams and the frictional torque to be expected is about two dyne-centimeters. Likewise, provided the rig is accurately aligned on the celestial pole, there is no component of earth's rate around the input axis: alternatively any component due to misalignment of the gyro on the mounting bracket will be constant irrespective of the rotational position of the spindle. This means that the torque motor current is only required to accelerate the gyro gimbal up to speed when the test starts and, thereafter, to cancel out parasitic torques such as flex lead torques, unbalances, and so on.

The record should therefore be a sine wave due to unbalance plus an offset from the centre on account of steady torques, both of which can be evaluated, and with a good gyro such a record is obtained together with exact repetition during each revolution after the first. The real value of the test lies, not in the information which it can provide about mass unbalance and steady torques, but in the information it can supply about repeatability, malfunctioning and/or erratic performance. In this latter respect it has proved invaluable. Repeat and/or other tests are however required to establish warm up repeatability and it is advisable to make a separate test to evaluate gimbal friction although its presence is generally indicated by "lumpiness" of the record. Fig. 16 shows the polar axis records of five different gyros.

If it is desired to perform the equivalent of a polar axis test on a two degree of freedom gyro, it is necessary to rotate it around its spin axis and make use of two complete separate closed loop servo systems and recorders.

After much experimentation the preferred method of measuring gimbal friction at the R.A.E. is still the method outlined earlier—by making use of a recorder with the closed loop test, by mounting the gyro firmly on a turntable and by rotating the turntable through a known angle—generally one minute of arc—as measured by an autocollimator. This method is preferred as being more searching than rotating the gimbal by injecting a current into the torque motor, experience having shown that the friction lag observed when the whole gyro is rotating about its input axis is generally about double that observed for a comparable rotation of the float caused electrically. Gimbal friction may be measured in any attitude of the gyro.

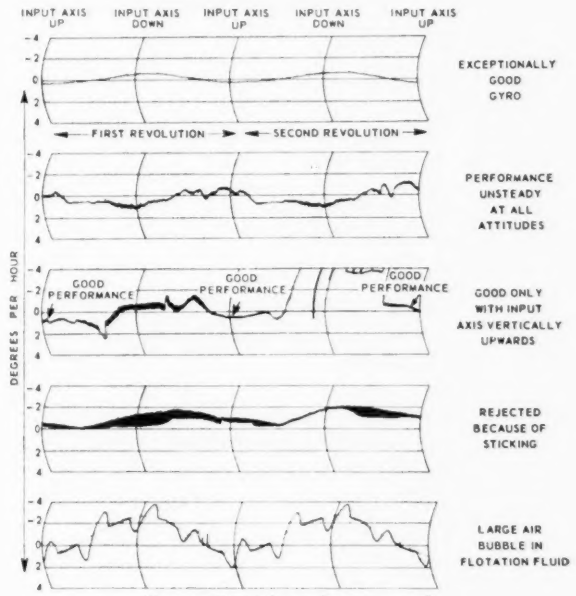

FIGURE 16. Polar axis test records.

4. Testing of Accelerometers

4.1. INTRODUCTION

Many accelerometers currently proposed for inertia navigation applications function by measuring the force necessary to give an internal mass the same acceleration as is applied to the case of the instrument.

As already mentioned they may be roughly divided into those in which the sensitive element rotates about some form of pivot or spring hinge, known as pivoted arm accelerometers, and those in which the sensitive element is constrained to move without rotation either in an arc of a circle or in a straight line by a system of supporting links, wires or ligaments, known as ligament supported accelerometers.

Clearly there are two fundamental methods of altering the force applied to the internal mass for calibration purposes; namely by altering the mass acted on by gravity as in the added weight test, and by altering the magnitude of the effective acceleration as in the tilting, vibration and centrifuge tests.

4.2. TILTING TESTS AS APPLIED TO "FORCE FEEDBACK" ACCELEROMETERS

The tilting test is one in which the accelerometer is calibrated against earth's gravity over the range -1 to $+1g$ by tilting the instrument from the horizontal, so that the resolved component of gravity along the sensitive axis is varied according to the sine of the angle of tilt.

It is convenient to consider this test first, since an examination of it makes clear the origin of some of the errors present in accelerometers.

Figure 17 shows the basic components of a force feedback accelerometer employing a parallel link support and tilted so that the sensitive axis is inclined at an angle α to the horizontal: the links being assumed to lie in the plane which contains the sensitive axis and which is perpendicular to the axis of tilt. (The sensitive axis is defined as the axis along which the accelerometer is designed to measure acceleration, as determined by the mounting surface of the case). Under these conditions the component of gravity along the sensitive axis is $g \sin \alpha$ and the cross component is $g \cos \alpha$.

The basic parts comprise a pendulous mass embodying an a.c. electrical take-off and an electrical restoring coil swinging in the field of a permanent magnet, together with an oscillator to energise the take-off, an a.c. amplifier, a phase sensitive rectifier, a stabilising network and a d.c. output stage, the whole forming a servo system. When the instrument is subjected to an acceleration with a component along the sensitive axis, the take-off produces a voltage as a result of the relative motion between the pendulous mass and its housing. The voltage becomes the servo error signal, which is first fed to the a.c. and then to the d.c. amplifier. The feedback signal is the d.c. current to the restoring coil which produces a force exactly balancing the force produced by the acceleration on the pendulous mass.

It can be shown that

$$I_\alpha = \frac{M}{k_e} \times g \sin(\alpha+\gamma) + B$$

to a high degree of approximation

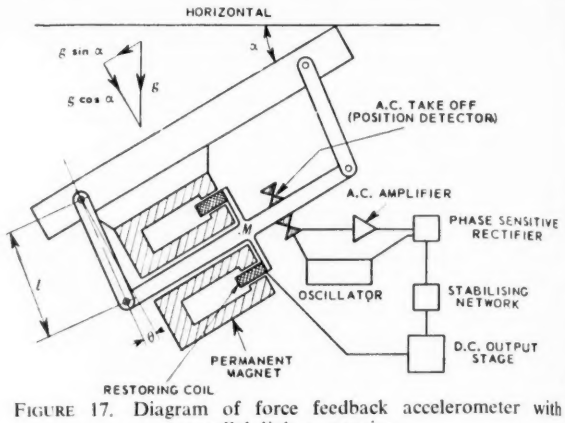

FIGURE 17. Diagram of force feedback accelerometer with parallel link suspension.

where I_α is the current in restoring coil at angle of tilt, α

k_e is electro magnetic transfer function of restoring coil

γ is axis error

B is the bias from all causes.

We can also show that B is approximately equal to

$$\frac{I_{90^\circ} + I_{270^\circ}}{2} \quad \text{or} \quad \frac{I_{0^\circ} + I_{180^\circ}}{2}.$$

The expression $\frac{I_{90^\circ} + I_{270^\circ}}{2}$ is the output current corresponding to an acceleration of $1g$ and is known as the scale factor. In order to eliminate errors due to bias forces or cross-coupling, the scale factor is, by definition, the mean of the numerical values of the output currents obtained with the accelerometer pointing vertically up and vertically down.

While it is convenient, when calibrating an accelerometer for linearity to adopt the concept of an axis error γ in our equation and to compare the output I_α with $\sin(\alpha+\gamma)$ plus a bias, the real significance is that we are introducing a cross-coupling term $M_0/k \cos \alpha \sin \gamma$ which can be seen by expanding the equation for I_α.

To achieve the necessary accuracy in a tilting test would involve measuring angles to seconds of arc, a feat which is by no means easy. Hilger and Watts, however, have quoted (in another context) accuracies for a 12 in. divided glass circle with double photoelectric reading microscopes, of 0·5 sec. of arc over small angles and 3 sec. of arc anywhere over the circle, which is the correct order.

An alternative method of measuring the angle of tilt is to use an autocollimator with one or more calibrated glass polygons. These latter are commercially available and the N.P.L. calibrations are to an estimated accuracy of ± 1 sec. of arc. Whichever method of measurement is used, extreme care will have to be taken to keep the datum of the apparatus fixed relative to the vertical.

4.3. THE ADDED WEIGHT TEST

The Added Weight Test, as its name implies, is a

test in which weights of accurately known mass are hung from or supported on the sensitive element.

With a pivoted arm accelerometer, the weights should clearly be hung from the centre of gravity but the difficulties are that its position is not known or defined to the necessary degree of accuracy, nor is it in general accessible. Nevertheless, by the exercise of ingenuity, it is possible to make the effective point of support of the added weight coincide approximately with the centre of gravity and, what is more important, be quite definite in position. The test can then be adequate to establish linearity over the desired range even if an absolute calibration cannot be established.

In the case of the link supported accelerometer the test is much easier to perform since, to a first order, the device is independent of the exact position of the centre of gravity but it is still necessary to maintain the sensitive axis accurately vertical, if a cosine error in calibration is to be avoided.

At the expense of some complication of the test apparatus, the down load of directly added weights can be supplemented by up loads obtained through the intermediary of a balance beam and the test can thus be extended to cover the full range from zero to full scale in either direction. Considerable experimental skill is required and the result is not an absolute calibration, but a calibration in terms of the local value of gravity as is the tilting test.

In one arrangement, access to the sensitive element was obtained by removing the normal end stop and by supporting the weight on an extension spigot slipped over the end of the sensitive element.

The spigot was pointed at its upper end where it supports the weights, so as to minimise parasitic torques and it was designed to sit squarely on the clamping nut and to be located by the plain section at the end of the sensitive element. It was made of hardened beryllium copper and the weights were made of brass, rhodium plated, and both were effectively non-magnetic. The weights ascend in intervals of about 9 grams and their absolute and incremental values were determined by double weighings to within about 0·2 milligrams with respect to the standard weights employed. The accelerometer was mounted in a rigid brass fixture and levelled to within 1 min. of arc. The design of the fixture was such that it could be turned either way up, so that the same weights could be used to simulate either positive or negative accelerations. Readings of output were taken with no weight added, with the spigot only added and then with each of the weights in ascending order. The accelerometer was then inverted and the readings repeated.

Since the weight of the sensitive element was not known beforehand it had to be determined by extrapolation and the method employed was to plot a curve of $\delta I/\delta W$. in milliamps per gram, against a horizontal scale of total weight based on an estimate of 18 grams for the weight of the sensitive element. This gave two groups of points one group on each side of zero and it will be seen (Fig. 18) that it is possible to draw a very fair curve through both groups. From this curve the best incremental scope applicable over the range $-1g$ to $+1g$ could be estimated and this value, in conjunction with the output readings at $-1g$ and $+1g$, enabled an accurate value to be obtained for the effective weight of the sensitive element. Using this value of weight and the incremental slope appertaining to $-1g$ to $+1g$, the theoretical value of the output was calculated for the accelerometer plus each value of added weight. Comparison with the observed output gave the accelerometer error corresponding to each value of simulated acceleration. These results are plotted in Fig. 19 and it will be seen that the errors under positive simulated acceleration are very different from those under negative simulated acceleration, possibly due to armature reaction.

Figure 20 shows the results of a test to determine whether there is a transient error due to the sudden application or removal of a large acceleration force. This test was performed by setting up the output measuring potentiometer to the correct output for the largest weight (or no weight), before the weight was applied (or removed) and taking readings of the output changes with time from the instant the load was in position (or removed). The curves, which are in terms of galvanometer deflection, show that there is an error of two or three parts in 10^4 which decays with a time

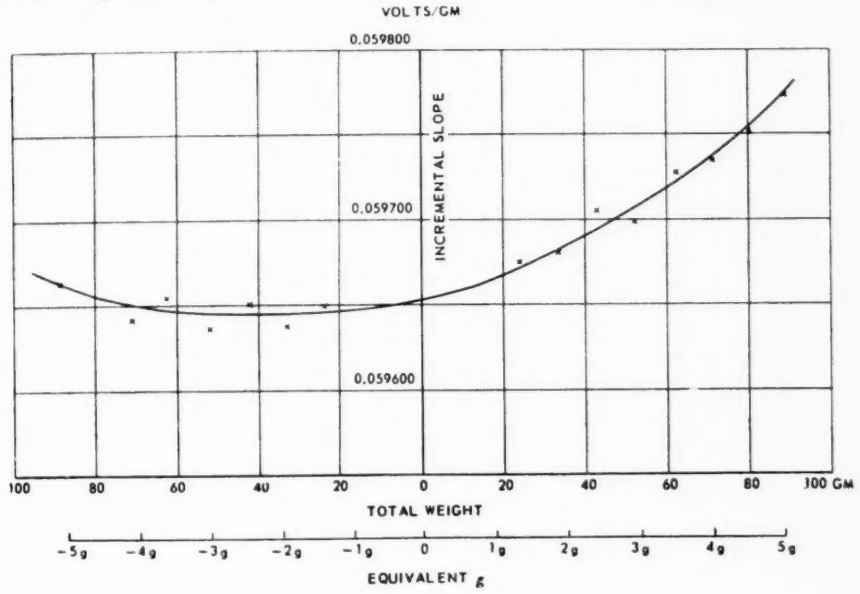

FIGURE 18. Added weight tests—incremental slope.

constant of about 30 secs.: it is suspected that this is a temperature effect due to the heating effect of the feedback current.

As regards the accuracy achievable by weighing, an N.P.L. Class A Certificate requires the weights to be adjusted to one part in 10^5, or one microgram, whichever is the greater, and the accuracy of certification is one part in 10^6. In international comparisons of the kilogram, the N.P.L. claim to have achieved an accuracy of one part in 10^9.

The criticisms normally levelled at the added weight test are:

(*i*) It imposes a concentrated load on the sensitive element whereas, in use in a uniform acceleration field, the sensitive element is subjected to a distributed load. Mechanical deflections on both the case and the sensitive element may therefore be different.

(*ii*) The addition of mass to the sensitive element will alter the characteristics of any servo loops and may upset their stability.

(*iii*) The addition of weights of magnitude several times the weight of the sensitive element, and at a point remote from the centre of gravity, may cause improper loading of the supporting ligaments and hence errors.

(*iv*) An accurate knowledge of the weight of the sensitive element is required.

5. Systems

5.1. GENERAL

So far we have been considering what might be described as pure inertia systems, that is, systems which depend solely on gyroscopes and accelerometers for their prime information. For flights of very short duration or such short horizontal travel that the earth may be considered as flat, we could devise a satisfactory and fairly simple accelerational system but for longer ranges we are forced to the "precessional" system employing the 84 minute pendulum.

I think that it is now apparent that the full realisation of a precessional system demands an accuracy of construction of accelerometers and gyroscopes which it is extremely difficult to obtain in practice.

Table I shows the magnitude of the position error which arises from a three gyro system with wander rates of one-tenth of a degree per hour. It will be seen that after a flight of 20 min. duration at 600 knots, the position is in error by almost one nautical mile.

There is, however, the possibility of reducing the stringent requirements of pure inertia navigation systems by combining their advantages with those of other navigation systems. These are usually known as mixed or "crutched" systems.

Inherently, any inertia navigation system has high accuracy for short periods, the accuracy deteriorating with time due to what might be called "drift" in the inertia references. Other forms of navigational aids, while possibly having good long term stability, suffer from errors due to random variations over a short period. There are also systems in which the navigation data is supplied at intervals of several seconds or may be interrupted by external conditions. By combining such aids with an inertia system it is possible to obtain the short term accuracy of the inertia system together with the long term accuracy of the auxiliary aid, which means that we can relax our demands for long term accuracy in the inertia components.

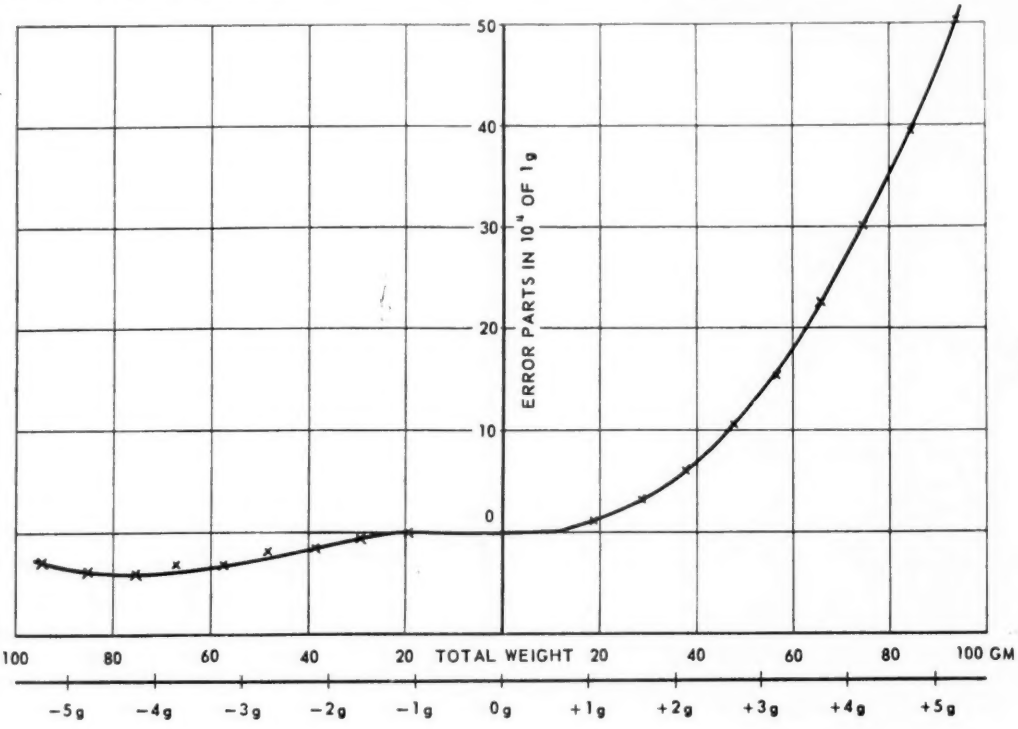

FIGURE 19. Added weight tests and total error.

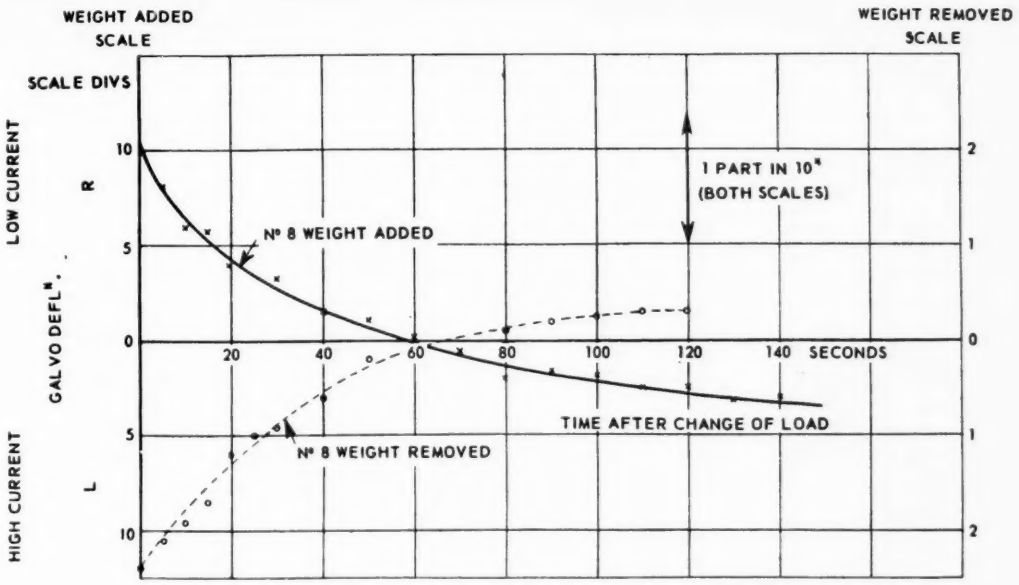

FIGURE 20. Added weight tests—variation of output with time.

Nevertheless we must remember that the use of auxiliary aids takes away the special self-contained property of the pure inertia systems, which we would only discard with great reluctance.

A pure inertia system computes ground velocity and displacement by integrating the output of two accelerometers. This information is fed back to the accelerometer platform through the servo system in order to maintain the platform perpendicular to the force of gravity. The two processes are inter-dependent; that is to say we need an accurate value of displacement to obtain an accurately stabilised platform.

It is clear therefore that any other method of obtaining an accurate value of the velocity or position of an aircraft can serve to monitor the values computed by the inertia system, and some examples will be described.

5.2. INERTIA AND ASTRO NAVIGATION

One other self-contained navigation system is by the use of observations of stars. A completely automatic celestial navigation system could be formed by making use of two automatic start trackers mounted on the inertia system's stabilised platform. With a suitable computer periodic fixes of the aircraft's position could be introduced as corrections to the co-ordinates determined by the inertia system. The inertia system would function alone during periods when the star trackers were out of contact because of cloud or for any other reason.

TABLE I

POSITION ERROR DUE TO GYRO WANDER

Time (*min.*)	*Distance* $V = 600$ *knots* (*n.m.*)	*Error components* $1/10^\circ$ *per hour wander rate in each vertical gyro* (*yards*)	$1/10^\circ$ *per hour wander rate in each azimuth gyro* (*yards*)
1	10	0·2	0·3
5	50	23	7
10	100	190	29
15	150	630	65
20	200	1500	120

A better arrangement, however, would be to use the automatic star trackers to provide stabilised spatial reference axes instead of gyroscopes: by a suitable feedback from an accelerometer it is clear that an 84 minute period Schuler pendulum system could be designed. Such a system might very well have gyroscopes as "smoothers" in the servo loops and if these were of the necessary precision, then the system would supply a memory for those periods when the star trackers were not locked on to the stars; during these periods the position errors would, of course, increase at a rate depending upon the precision of the components but these cumulative errors would be eliminated when the trackers regained their lock.

5.3. INERTIA NAVIGATION AND DOPPLER

The Doppler system of navigation is now well known. By using the Doppler shift in frequency which occurs when radio signals transmitted by an aircraft are reflected back from the ground, it is possible to produce a system which measures ground speed and drift angle. When this information is fed, together with heading, to a suitable computer, it is possible to indicate continuously the position of the aircraft. The system, however, is subject to errors when the aircraft is flying over mountainous country, and over the sea under certain sea states, and it is the usual practice to include a "memory" system which can be switched on when the system is thought to be unreliable: its accuracy is also dependent on the accuracy of the heading information fed into the computer. We can replace this "memory" system by an inertia system. The Doppler radio could, in fact, be regarded as the prime producer of velocity

and the accelerometer-measured velocity from the inertia system would serve to smooth out the momentary errors or "noise" and also take over for relatively long periods when the Doppler system is out of action. During war-time, for example, it might be undesirable to operate the Doppler radio except for short bursts of transmission at irregular intervals.

5.4. INERTIA NAVIGATION AND RADIO

There are various methods of position fixing by radio. These differ from the previous methods described in that equipment on the ground is necessary in addition to equipment in the aircraft. The previous two are, of course completely self-contained within the aircraft and independent of ground facilities.

Almost all such systems are liable to jamming and to radio interference to some degree or other and have limitations on accuracy in certain positions and at certain ranges. In some cases, too, the data rate is low, *i.e.* the aircraft fix is obtained only at discreet time intervals. It is possible, however, to use the position information obtained from such interval radio aids to "re-set" an inertia system and the inertia system would carry on until the next fix is obtained.

5.5. GYRO ACCURACIES

We have to remember that the importance of the mixed systems in relation to inertia navigation is the relief it offers to the standards of component accuracy. The need for accuracy is related to the square, or in some instances the cube, of the time of flight, and we have seen that the aim for pure inertia systems, for flights of long duration, is to attain wander rates of small fractions of a degree per hour. Once, however, we think of combining inertia systems with other systems, our standards are reduced so that we can talk in terms of one or two degrees wander per hour, which makes the problem very much more simple.

6. Future of Inertia Navigation

Although one must give the highest praise to the theoreticians, the scientists and the designers who, in recent years, have achieved such great improvements in gyroscope and accelerometer performance, there is no doubt also that these have only been made possible by extremely high standards of engineering, which are very difficult to attain in even moderate scale production. Moreover, the maintenance of that original precision, both during storage life and in service, will give rise to prodigious difficulties. As we have seen, we are dealing with alterations in physical dimensions of the order of less than the wavelength of visible light or the crystal lattice distance of most of the materials we are using. Such precisions are generally only attained in very high grade optical equipment, normally kept in laboratories, under undisturbed conditions of temperature pressure and movement. Compare this with the life of a piece of military equipment which, after leaving the factory, may be sent to any part of the world, stored indefinitely, and then will be required for use at the shortest notice; and probably operated by inexperienced troops!

Of course, difficulties like this can be overcome, but only at great cost in both effort and loss of flexibility. It does seem therefore that the field of application of pure inertia navigation systems will be in vehicles which have only short times of flight. Most of these seem, at present, to be military in nature, but then it is unfortunately the case that most developments which are so costly in effort as to be beyond civil or industrial resources, owe their existence to a military interest; usually more useful applications follow and it may be that in future these systems or their components, into which so much thought and effort has been poured, may play their part in the advance of civilisation.

The great advantage of inertial systems is their independence of outside sources of information which might be interfered with by some external agency. This, indeed, is the very property which has provided most of the stimulus for their development, but again I am bound to feel it is mainly a military stimulus. Navigators may point out meteorological or other conditions which will interfere with orthodox types of navigational equipment and while this is admitted, there are now so many different types of radar and radio aids to navigation which can be installed in the same aeroplane that I do not feel airline operators will be attracted by the expensive and delicate components of inertia navigation systems.

For similar reasons, I do not believe that the mixed systems, such as I have described earlier, will have any widespread civil application, although there are clear military advantages in an aircraft navigational system which will continue to operate accurately and precisely for restricted periods while contact has been lost with external sources of information.

There are, however, important by-products of these researches. Many of the techniques and design methods which have been used in the development of the super precision inertia components can be applied to the similar but much less precise equipment used in normal pilotage, with a consequential increase in accuracy and reliability. This is, indeed, some reward for the effort which has been expanded.

In conclusion, therefore, my own view is that the future of inertia navigation equipment logically lies in its application to those military problems involving short times of flight, such as guided and ballistic missiles. If and when we embark on space travel then similar equipment may be necessary, but I am not one of those who regard such explorations as likely in the near future!

I would like to thank Mr. Whitaker and Mr. Grimes, both of the Ministry of Supply, who have helped me in the preparation of this paper.

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The Man-Powered Aircraft

A Design Study

by

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SUMMARY—An appreciation is made of recent experimental and project work of relevance to the design of a very light weight aircraft capable of being taken off and flown by two men. The emphasis is on the aerodynamic problems involved. It is concluded that, although there are many difficulties still needing attention, all the information supports previous assertions that flight by muscular power alone is possible. The merits and de-merits of a particular projected design are studied, and its performance and stability assessed.

1. Introduction

Since the publication in 1956 of a provocative assessment of the possibilities of man-powered flight[1], and some tentative calculations by the author[2], a good deal of further thought and some experimental work has been devoted to proving its possibilities. Now that a Committee has been formed to promote the project, under the Chairmanship of H. B. Irving, it seems appropriate to summarise the work undertaken, to act as a guide to future investigators, and as persuasive evidence to convince those who are still sceptical of the possibilities of man-powered flight. It is hoped that, most of all, it will provoke constructive criticism which the author would be glad to receive.

Without intending to imply that it has any particular merit, we consider here only a *fixed-wing* version of the aircraft. A man-powered helicopter or even an ornithopter has possible advantages but would require quite separate and detailed study. Similarly we have not considered the employment of aerostatic lift.

2. The Power Generated by Man

We all know that one can work harder for a short period than for a long one but for some strange reason the power output of man and the way in which power decreases with increasing duration of exercise seem not to have been seriously studied by physiologists. If such studies have been made they have been lost from the stream of physiological knowledge. The only known systematic studies of the subject have been by engineers[3], [5].

In Ref. 3 details are given of measurements of the air resistance of cyclists in the posture adopted in racing, and since this source provides the bulk of the resistance to their motion on a hard track, these results can be used to provide reliable indications of the power output of record-breaking cyclists, as shown in Fig. 1. Over durations between one minute and half-an-hour the National records approximate to a level 0·49 h.p. output plus the ability to release an extra 13,000 ft. lb. of work at will; this corresponds with the concept that a man's power output depends in part on his oxygen intake (varying in proportion to time of effort), and for the rest on his "oxygen debt."

These results are compared in Fig. 2 with those of Ursinus[6], and fragmentary data from other sources, relating to other methods of power production. An opinion on the comparison has been sought from Dr. D. R. Wilkie of the Physiology Department, University College, London, who has made an extensive search of the literature and uncovered further results not shown in Fig. 2. His conclusion is that, although the cycling action permits large power outputs at all durations and is mechanically simple, nevertheless the legs cannot by themselves utilise to the full the short-term energy stores of the body. In amplification of this he writes[14]:—

> "The muscles are a machine for transforming chemical into mechanical energy. The chemical energy comes ultimately from the oxidation of a carbohydrate, glycogen, to carbon dioxide and water. All the chemical processes take place at constant temperature *i.e.* the energy does not appear at an intermediate stage as heat, so muscle cannot profitably be compared to a heat engine. In spite of the complexity of the chemical processes involved, muscle achieves an efficiency of 20 to 25 per cent under favourable conditions.
>
> "Ultimately the energy production of muscle thus depends on an adequate supply of oxygen, which must be absorbed at the lungs and transported by the bloodstream. These organs have a limited capacity which in turn gives a limit to the steady-state energy production; healthy young men can absorb at the most about four litres of oxygen per minute while the maximum recorded (in an Olympic athlete) was about 5·4 l./min. These oxygen consumptions correspond to power outputs of about 0·4 and 0·54 horse power respectively."

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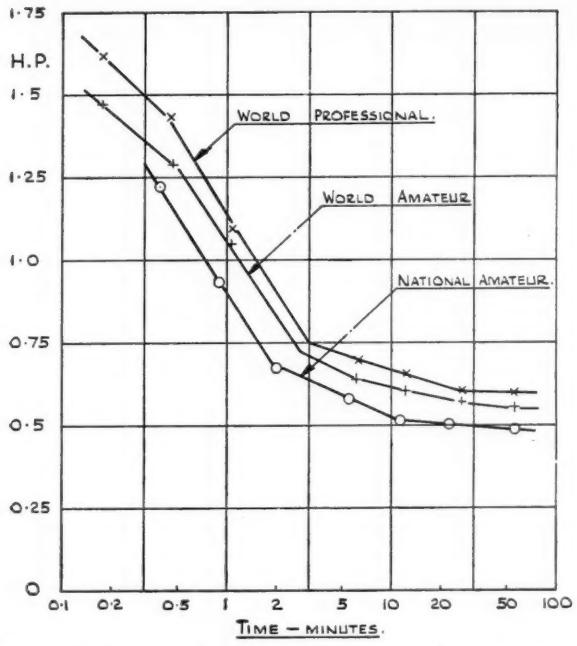


FIGURE 1. Powers to be achieved by a cyclist of average size to equal various record performances.

The latter figure agrees reasonably with the data of Fig. 1, indicating that the legs alone can generate the full-steady-state work production. Wilkie presents evidence, in support of the other figure, that training can add 20 to 30 per cent to an individual's power output. He goes on:—

> "The much greater power available for shorter periods is only obtained by going into "oxygen debt," that is, obtaining energy from the hydrolysis of chemical stores rather than from their oxidation. In order to replenish these stores at the end of exercise an extra amount of oxidation must take place—the "oxygen debt" must be paid back during recovery. For brief efforts the power output depends on the bulk of muscle that can be brought into use and on the availability of hydrolysable chemical stores. The maximum "oxygen debt" that can be accumulated is about 20 litres, equivalent to 60,000 ft.lb., and takes 2-5 minutes to be mobilised."

We have seen that less than a quarter of this "debt" is mobilised in cycling, but Wilkie finds that cycling with hand-cranking can nearly double this figure. In the only experiment on a cycling athlete, peak output was increased from 1·28 h.p. to 1·89 h.p., whereas the subject of Ursinus's tests, H. Gropp (who was not an athlete), was able to match Reg Butler's cycling performance at 15 seconds, by the help of hand-cranking. Wilkie reports:—

> "Four different types of machine for combined arm and leg movement were thoroughly investigated by Ursinus, who concluded that simple rotation of a 17·5 cm. crank by hand and foot was the most effective arrangement as well as being mechanically the simplest. Ursinus also established the optimum speed for each duration of effort, the correct phase relationship between arm and leg and the scope and limitations of postures ranging from lying on the back through normal sitting to lying face downwards."

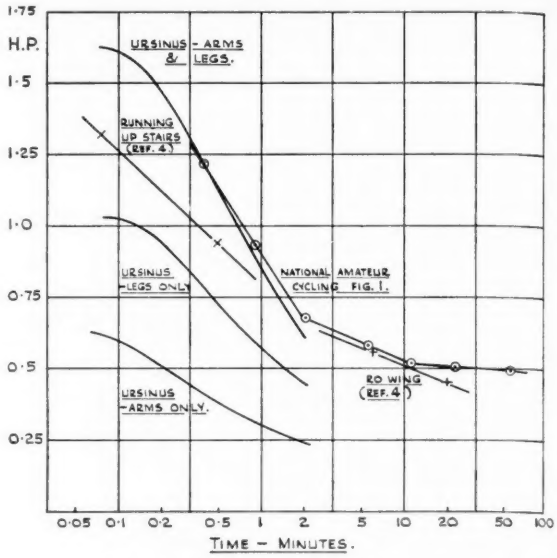


FIGURE 2. Various estimates of man's power compared.

Thus there seems to be little doubt that performance for the first two or three minutes can be substantially improved by combining arm and leg cranking.

For the purposes of the present study, it has been assumed however that the cycling action would be used, on the grounds (*i*) that it would enable one easily to find those athletes who excel in this particular exercise, (*ii*) that the performance of cyclists is well proven, (*iii*) that at least the pilot's hands must be free to control the aircraft, and (*iv*) that durations of flight would be long enough to make little difference if hand-cranking were adopted. It subsequently transpires that the last named is an inappropriate premise, and that a short duration burst of energy is what is required for flight. Hence one can expect some considerable improvement if one member of a two-man team derives additional power by hand-cranking. The actual figures used are 90 per cent of those quoted in the opening paragraph of this section—as of course it would be unreasonable to expect a record-breaking level of achievement on each attempt. Such figures are supposed relevant (as explained in Ref. 3) to a man of average size (height 70 in. and weight 150 lb.); insufficient is known as yet to deduce the relation of the power output to the weight of various individuals.

Apart from selection, training, and so on, only two ways have been suggested for increasing power output: warming the whole body by diathermy, or administering oxygen. The former would be effective but might be dangerous, the latter would certainly be of great

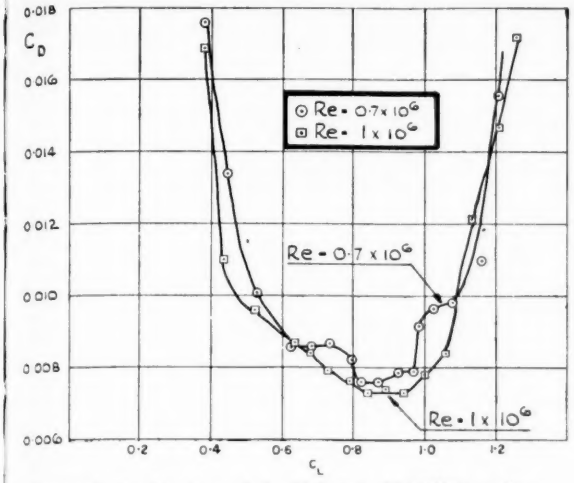


FIGURE 3. Profile drag of the N.A.C.A. 65A(10)12 section.

benefit if flights of five or more minutes' duration are envisaged, for oxygen administration is known to improve the steady-state power production. For durations of two minutes or less it may (Hill, Long and Lupton) or may not (Krogh) be effective. Curiously, 66 per cent oxygen is more effective than 100 per cent oxygen, but the reason is not understood.

3. Choice of Wing Section and Plan Form

The section (lift/drag) ratio is a convenient guide to its merit in the application envisaged, but the aircraft wing is expected to operate at a mean Reynolds number of 0·9 million and systematic information on sectional characteristics in this range is sparse. The highest (lift/drag) ratios recorded at this Reynolds number were thought to be those measured[6] on a N.A.C.A. 65A(10)12 section—a low drag wing with 6 per cent camber—and the lift-incidence, lift-drag polars and pitching moment characteristics of this wing are shown in Figs. 3 to 5. This section data has been used in the project analysis appearing later, but the camber is possibly excessive, because almost as good a (lift/drag) ratio has been achieved[7] on a section with a design lift of 0·6 (as shown in Fig. 6), and the lower operating lift coefficient, the lower C_{D_0} at low C_L (*i.e.* at higher speeds), the smaller value C_{M_0}, and the more easily constructed shape of the under surface are all minor and indirect advantages of its use.

In the other direction, more recent information has suggested that the low-drag profile designated FX05-H-126 by Wortmann[8]—which has a (t/c) of about 13 per cent, a camber of 5 per cent, a nearly flat undersurface, and a reflex curvature on the upper surface, extends the envelope of the drag minima, so well indicated by Fig. 6, up to a C_L of over 1·1. From the performance point of view alone this section is even more attractive, though working so near the stall might be a disadvantage.

The optimum operating lift coefficient of the N.A.C.A. 65A(10)12 section is found later to be around

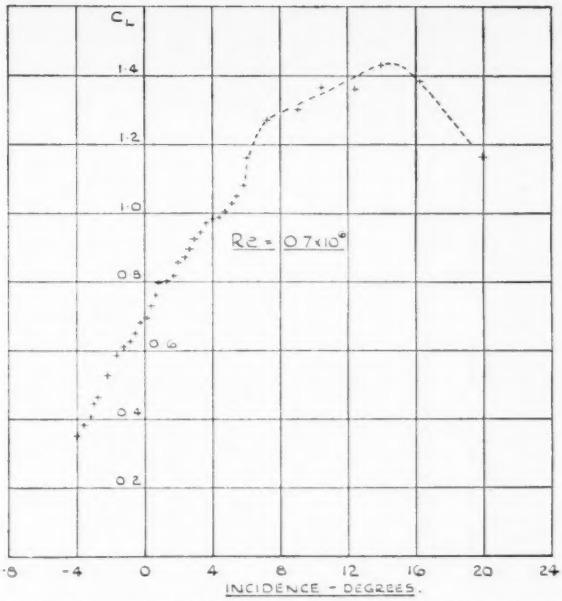


FIGURE 4. Lift-incidence behaviour of N.A.C.A. 65A(10)12 section

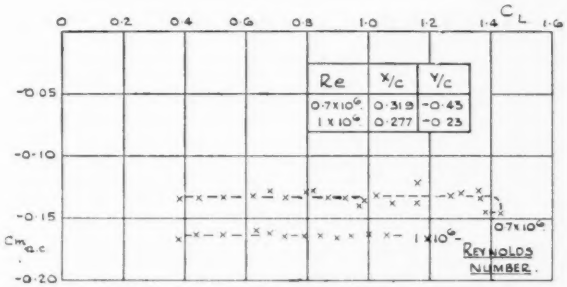


FIGURE 5. Pitching moment coefficient resolved about the aerodynamic centre position.

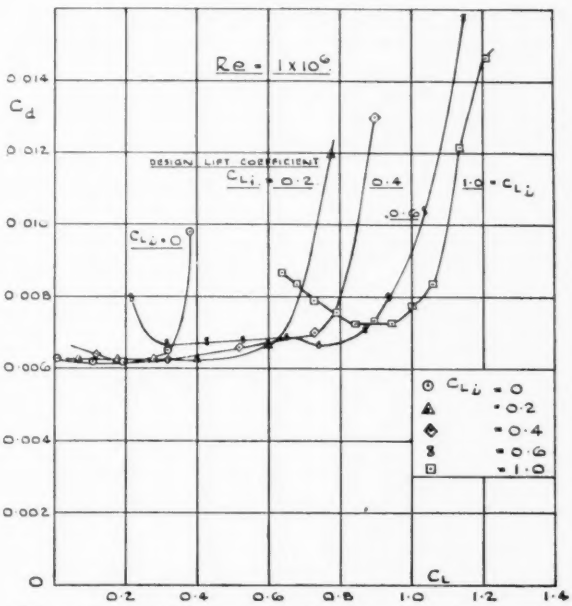


FIGURE 6. Profile drag of N.A.C.A. 64 series wings of various camber compared with that of 65A(10)12. (N.B. 64-series wings might be expected to produce about 0·0005 more in C_{D_0} than the 65-series at the Reynolds number quoted.)

0·94 (at a speed of 50 ft./sec.) and as it stalls at about $C_L = 1{\cdot}45$ at 10° greater incidence, gusts of more than about 6 knots could be troublesome. This, more than anything else, shows that man-powered flight is a calm weather sport, as the reserves of power and height to recover air speed are virtually non-existent.

It is not known how sensitive such low-drag sections may be to the maintenance of the correct profile at this scale, and it would be valuable to make comparative tests on practical construction wing-sections to discover this. Certainly it would seem that the Reynolds number is so low that surface roughness is of no concern. The wing profile drag transpires to be about 30 per cent of the total resistance, so that development of slot suction wings can be seen to be of particular advantage.

The main effect of plan form is on the induced drag which is an even larger proportion of the total drag—nearly one half. Thus savings here are of great importance. The construction of a wing with anything other than uniform taper was considered too elaborate for an initial survey. For an aspect ratio of 20, the minimum induced drag is achieved with a 4 : 1 taper ratio providing an induced drag factor of 1·02 (Fig. 7). In the subsequent project analysis a taper of 5 : 1 was selected under the erroneous impression that the saving of wing weight might compensate for the increased drag. The span loading of this wing is shown in Fig. 8, and it suggests that either washout or greater taper near the tips would be of advantage; of even greater importance—if some more elaborate plan form were considered—would be a local increase in chord near the root to counteract the adverse effect (on induced drag) of the negative lift on the tail.

Ground effect will later be shown to reduce greatly the induced drag. It is not exaggerating too much to say that the success of the project hangs on the reality of the correction adopted for this effect. That assumed in the subsequent project analysis is taken from Ref. 9 and is shown in Fig. 9. This figure refers to elliptically loaded wings to which that chosen in the present work only very roughly approximates.

A case can be made for the provision of sweepback on aerodynamic grounds, but it has been considered unwise owing to constructional difficulties and lack of certain knowledge of the likely effects.

4. Method of Wing Construction and Structure Weight

Structures of the dimensions of the envisaged aeroplane are of course common in glider design, where however the design loads are much heavier than any we need anticipate. Indeed it is quickly established that present day gliders could not be sustained by man-power alone, so that we must hope to gain something by sacrificing some measure of airworthiness. Indeed there would seem no reason to suppose that the pilot would want to apply more than tiny increments of acceleration, but he might inadvertently do so if he lost control. In view of this a limit load of $1{\cdot}5g$ would seem adequate. Conventionally a factor of 1·5 is applied to this, but in view of the possibilities of inaccurate stressing a total factor of 3 has been assumed here. (Later on, when more careful stressing is used, it might be safe to come down to a factor of 2·5.)

It seems advisable to design the wing with a rigid skin, at least until we know more about the aerodynamic effects of the distortion of fabric or paper covered panels. After several attempts to evolve a satisfactory method of construction combining fibre glass and foamed plastic, P. Jeffery (Design Dept., College of Aeronautics) decided that the more conventional $\frac{1}{32}$ in. thick birch ply covering is the lightest practical material. A rib pitch of 9 in. was considered necessary to support this, with ribs of conventional spruce construction. A single box spar at 40 per cent chord with variable boom thickness over the span, designed to carry an all-up weight of 490 lb. with an ultimate factor of 3, forms the largest item in the weight breakdown which is given for a specimen wing plan form in Table I, and the details of its construction are sketched in Fig. 10. The effects of

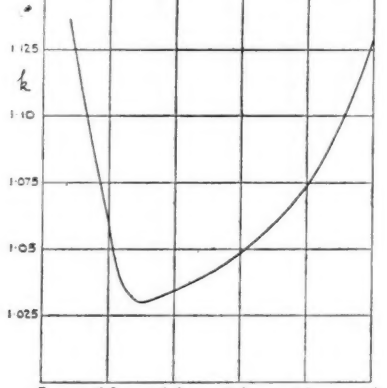


FIGURE 7. Variation of induced drag factor with taper ratio for wing of aspect ratio 20.

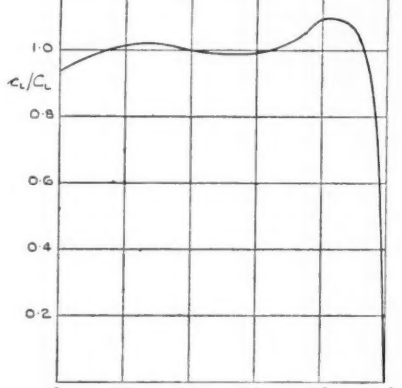


FIGURE 8. Variation of sectional lift coefficient on wing of aspect ratio 20 and taper ratio 5:1.

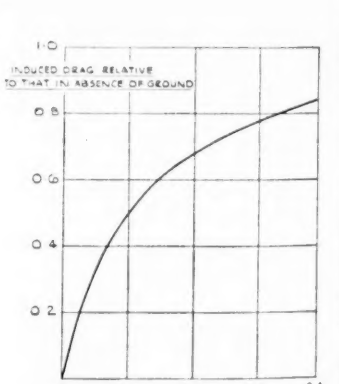


FIGURE 9. Ground effect on induced drag. (For wings of elliptic loading.)

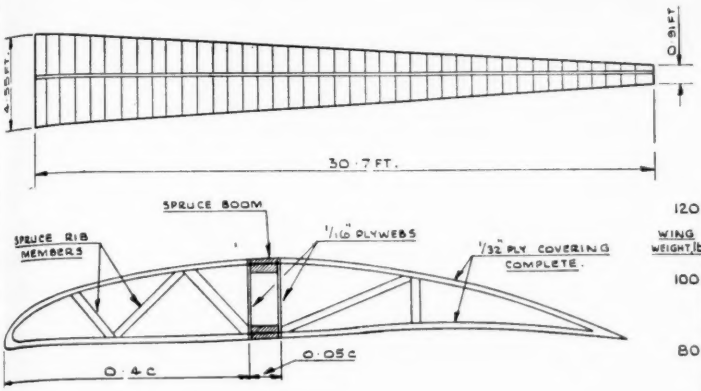
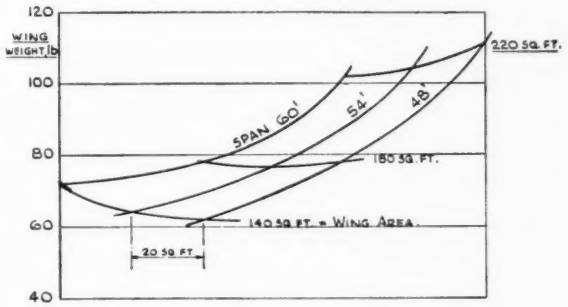


FIGURE 10 (*left*). Illustrating the assumed method of wing construction.

FIGURE 11 (*below*). Carpet plot of wing as affected by area and wing span.



variation in chord and span on wing weight is shown in Fig. 11.

No allowance has been made for the provision of control surfaces; indeed their form and size has not yet been established to enable this. Likewise, no careful consideration has been made of the other items of structure weight but tentative estimates of these are given in Table II, referring to the projected layout envisaged in making the performance calculations later (*see* Fig. 14). It has been considered that all the surfaces should be rigid to preserve a low-drag form; owing to its relatively small size, the tail-plane would probably have to be shaped from solid birch; the propeller blades are also supposed solid. Details of the other items will emerge from the following paragraphs.

5. General Layout

The fuselage layout presents a considerable problem; we shall suppose that it is necessary to accommodate two men in tandem in the cycling position, two men being the least required—one to ensure power continuity while the other is temporarily abstracted by controlling the aircraft—and three or more men seeming somehow to invite derision—although it may well be true that an "eight" is the optimum in flying as in rowing! As it is considered essential to the spirit of the enterprise for the aircraft to be propelled from rest to the take-off speed, wheels with low capacity high-pressure tyres would also need to be contained within it. One is therefore presented with the problem of fairing in what amounts in its geometry to two men on a tandem bicycle: the frontal projection being tall and thin immediately suggests that the form of this part of the fuselage should be like a fin, with a thickness/chord ratio adjusted to provide minimum drag for the given frontal area. Experience of wing shapes of higher aspect ratio and at higher Reynolds numbers suggests an optimum around $(t/c)=0{\cdot}2$, although as the fuselage drag is only 6 per cent of the total, such considerations are not important, and the precise figure would best be dictated by lateral stability characteristics (Section 9) which such a large fin shape severely modifies.

A high wing would seem to be more practical—to avoid impact of the wing tips on touch down or take-off, and the inevitable strengthening of the wing this contingency would require. In any case the relative loss in the alleviation of induced drag by ground effect is probably more than compensated in the high wing position by the reduction in wing-body interference, as contact between the upper wing surface (where there are strong adverse gradients) and the fuselage would

TABLE I

WING WEIGHT

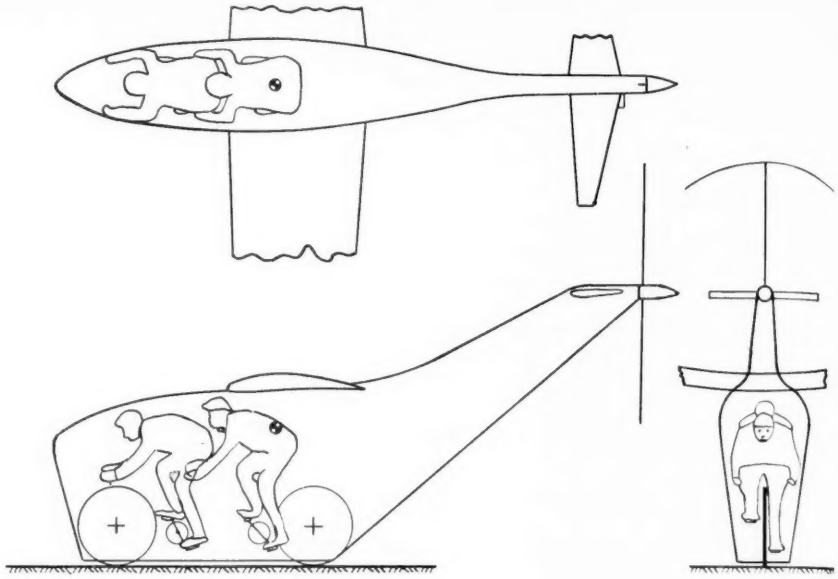
			Percentage
Spar boom top	14 lb.		18
Spar boom bottom	9 lb.		12
Spar web	6 lb.		8
Total spar		29 lb.	38
Covering		25 lb.	32·5
Ribs		18 lb.	23
Miscellaneous		5 lb.	6·5
TOTAL WING WEIGHT		77 lb.	

TABLE II

WEIGHT BREAKDOWN

		Percentage of empty	*total*
Wing (*see* Table I)	77 lb.	45	16
Fuselage-fin	40 lb.	23	9
Propeller	10 lb.	6	2
Tailplane	10 lb.	6	2
Transmission	10 lb.	6	2
Wheels	10 lb.	6	2
Controls	5 lb.	3	1
Contingency	8 lb.	5	2
EMPTY WEIGHT	170 lb.		36
Crew	300 lb.		64
TOTAL WEIGHT	470 lb.		

FIGURE 12. Fuselage and fin layout.



lead to trouble. In a low drag layout the major contribution to profile drag could arise from interference effects, and so it would seem essential to sweep the maximum thickness position of the fin-shaped fuselage backwards at its tip, so that the junction at the wing is between two surfaces supporting favourable pressure gradients. It would be helpful as well to give the fuselage a N.A.C.A. low drag section (say the 66-021) to stimulate these favourable gradients.

Such a side shape for the fuselage leads naturally to its extension into a true fin aft of the c.g., as shown in Fig. 12. This layout is made all the more inevitable by the fact that it can conveniently accommodate the pusher-propeller necessary to avoid spoiling the flow over the wing, which hazard would result from a conventional tractor position. No close study of propeller design has been made, but use of typical pre-war charts of low scale data suggests that a large diameter disc is essential for a reasonable efficiency (Fig. 13 from Ref. 10). To avoid fouling the blades with the ground, such a pusher must be placed high above the c.g., *i.e.* at the top of the fin. The elaborate transmission system necessary if the fuselage were connected to a true fin by a boom, let alone the difficulties of fairing the fuselage to the boom, inhibit consideration of a layout more conventional than that shown by Fig. 12. We shall see later that the high propeller position is troublesome to the longitudinal stability and control of the aircraft, but it appears to be an essential feature, and indeed appeared on all the man-powered aircraft designs of the pre-war era.

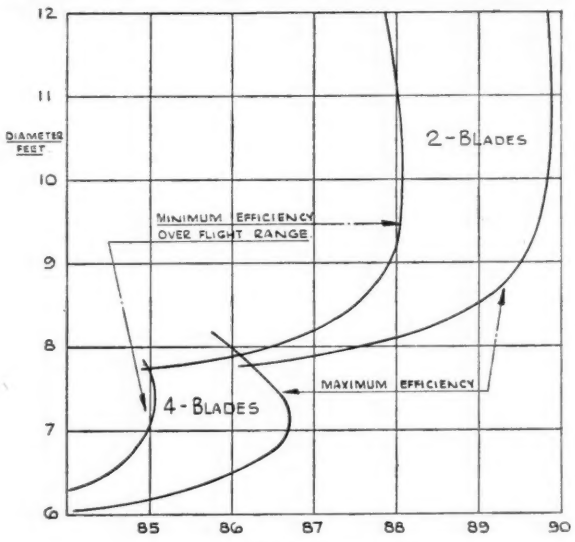


FIGURE 13. Variation of propeller efficiency with blade diameter.

The length of the fin extension above the fuselage should be kept as short as possible, the critical interest lying in providing just an adequate tail arm rather than a large one (*see* Sections 9 and 10). The general arrangement shown in Fig. 14 has been assumed for initial assessment and its stability characteristics are discussed in later Sections, where certain important changes are suggested.

A summary of the leading geometrical features of this layout appears in Table III. It involves an optimised wing span and chord based on an examination of the effect of these variables on the minimum power needed to maintain level flight at an altitude of 35 ft. The

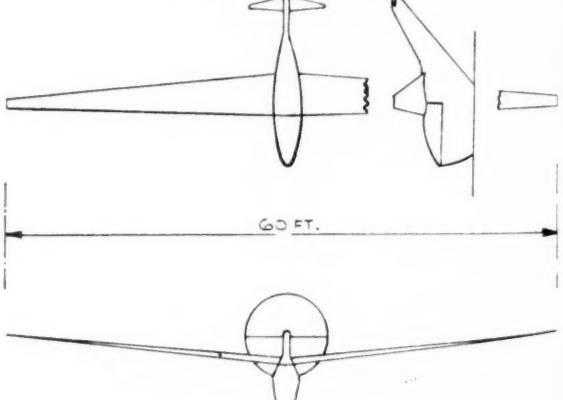


FIGURE 14. General arrangement of projected aircraft. For main dimensions—*see* Table III.

TABLE III

GENERAL DETAILS OF PROJECTED AIRCRAFT

Wing

Section	N.A.C.A. 65A(10)12	Taper ratio	5:1
Area	168 ft.²	Aspect ratio	21·4
Span	60 ft.	Dihedral	5°
Mean chord	2·8 ft.	A.C. position	0·28*c*

Line of zero sweep at 0·28*c*, 0·29 ft. ahead of *c.g.*
Chord line parallel to ground line.

Tailplane

Section	N.A.C.A. 65-618	Mean chord	1 ft.
Area	8·6 ft.²	Taper ratio	5/1
Span	8·6 ft.	Aspect ratio	8·6

Tail arm from wing a.c. 10 ft.
Tail height above c.g. 4·2 ft.
Tail volume 0·183
Tail load, power on 17 lb. (negative lift) $C_{L_t} = -0{\cdot}66$
power off 10 lb. (negative lift) $C_{L_t} = -0{\cdot}38$
$d\epsilon/d\alpha = 0{\cdot}16$ without ground effect.

Fuselage-fin

Section	N.A.C.A. 66-021
Height	5·5 ft. at wing junction 8·75 ft. maximum at tail
Side Area	67 ft.² total 13 ft.² above wing t.e.
Length	18·6 ft. to propeller disc
Ground Line	4·3 ft. below c.g.

Wheels

1·25 ft. radius. Ground contact 5 ft. ahead of } c.g.
1·25 ft. behind }

Propeller

Blades	2	Height above c.g.	4·2 ft.
P/D	1·6	Diameter	8·3 ft.

optimum span is found to be 60 ft. and the optimum wing loading a little less than 3 lb./ft.². However, the optimum span is sensitive to taper ratio, and it has already been suggested that this should be changed from 5 : 1 (assumed) to 4 : 1. Again, subsequent performance evaluation suggests that if the men are not to use assistance in the take-off, flight would be much closer to the ground where the reduction in induced drag would make it worthwhile to increase the wing loading. It is, of course, a nice point whether one would want to design the first aircraft to provide the easiest means of achieving man-powered flight, pure and simple, or whether one would prefer to look ahead to the use of some form of assistance which would lead to man-powered flight at higher altitudes (*see* Section 8). In any event a reassessment of the optimum wing loading and span now seems necessary.

6. Transmission and Propeller

Little complication would be involved by providing a drive to the propeller through the wheels so that during take-off the advance ratio of the propeller could be held at a constant value. As a wheel-drive is a much more efficient means of propulsion one would want to choose the gearing and this advance ratio so that only towards the end of the take-off run would the propeller take over full propulsion. If a free wheel is provided, the propeller could then achieve faster r.p.m. at this ground speed—and so generate higher powers for any further acceleration. (A free wheel would also be needed if the wheels are used on landing, although an extensible skid might be more appropriate.) The alternative would be a variable pitch propeller which would be far less efficient.

Though no serious difficulty is anticipated, little thought has yet been devoted to the transmission system to the propeller, which is over a length of about 12 ft., and must provide for the flight range in operating r.p.m. shown for a typical design in Fig. 15. A hydraulic system would be appropriate in view of the ultimate necessity of turning the torque axis through 90°, but its efficiency might not exceed about 92 per cent, and its overall weight would not be substantially less than that of a light-alloy chain and bevel or worm drive providing much higher efficiency.

Likewise, work needs to be done on evolving a satisfactory propeller. The operating conditions suggested by the later work are two power levels—cruise and maximum power—differing by a factor of 2·5:1—while the flying speed range is very small. The advance ratio and pitch would want to be chosen so as to provide as high an efficiency as possible in both conditions; at maximum power, the advance ratio used would therefore be slightly less than that for maximum efficiency. The data used in the project analysis subsequently quoted are based on the old charts of Ref. 13, as they are more likely to be relevant to the blade Reynolds number of about 4×10^5 involved, than more recent data. On the other hand, the propeller's range of operating conditions is to be relatively small and careful blade design might well provide even higher efficiencies than those assumed.

In considering the transmission and propeller design it is worth noting that an increase of one per cent

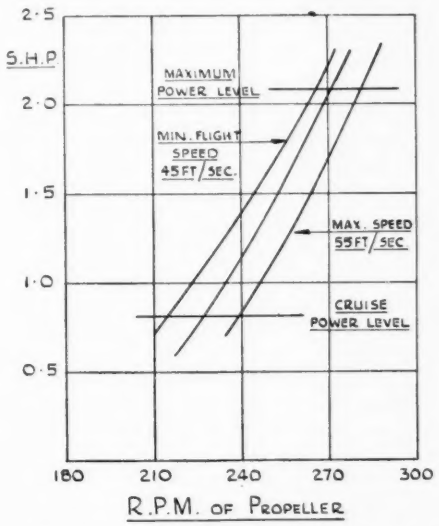


FIGURE 15. Operating conditions of propeller. (Tentative.)

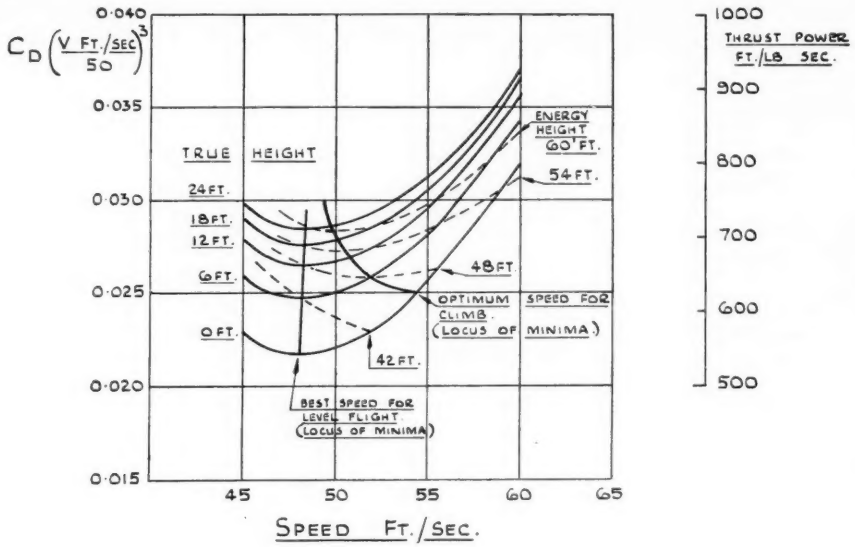


FIGURE 16. Power coefficient required for projected aircraft.

efficiency is worthwhile, provided it can be achieved at the expense of no more than $3\frac{1}{4}$ lb. of weight.

7. Take-off and Performance

One is forced to admit for a number of reasons that acceleration during the ground run is a far more efficient means of power absorption than flight, so that one would want to delay take-off at least until the optimum climbing speed is reached (which is about 55 ft./sec. at sea level, *i.e.* at $C_L = 0{\cdot}78$). On the other hand it would be as well to choose the ground attitude of the aircraft so that (with both wheels grounded) some lift is developed on the wings and the proper balance is struck between the induced drag produced and the alleviation of rolling friction: careful determination of this attitude has not been made but it would appear to correspond to that required for take-off. As the speed builds up, the lift on the wings takes the weight off the wheels, and progressively more of the torque is absorbed by the propeller.

Given the power capabilities of the two men as an ability to maintain indefinitely a minimum ("cruise") level of power output, plus a potential to do a certain amount of extra work, it can be shown that the greatest *height* of flight will result from: —

(*i*) accelerating from rest on the ground at the cruise power level until the speed steadies (found in the worked examples to be at about 45 ft./sec. after a run of 900 yd. lasting 80 sec.) by which time the propeller has taken over full absorption of the power input—the wheels having freed;

(*ii*) increasing power to as high a value as possible (taken to be $1\frac{1}{4}$ h.p. in the worked example) and accelerating to the take-off speed (55 ft./sec. reached after 10 secs. in a travel of another 150 yd.); and

(*iii*) continuing at maximum power to climb on a path giving maximum rate of increase of energy height (Fig. 16), which involves a progressive loss in speed.

Upon exhaustion, the crew would return to the cruise level of power output and continue to ascend until nearing the stall.

Physiologically, this method of power output would be quite appropriate, the initial period of cruise power providing a " warm up " and the return to it after more violent exertion being a not uncommon habit of athletes. Whatever the total flight duration, it would be appropriate to employ sprint cyclists used to short, sharp bursts of power generation.

For the greatest flight *duration* the best flight programme is merely to take off and fly at the minimum power speed (48 ft./sec.) as close as possible to the ground; the power needed is then not much more than that of the cruise condition but would depend on the actual altitude.

To determine the heights and durations possible, it has been assumed that the air crew are of average size and develop 90 per cent of the National record level (0·49 h.p. plus 13,000 ft. lb.), that 4 per cent of this is lost in the transmission to the propeller, and that the efficiency of the latter is 87 per cent. In the ground run the thrust supplied by the wheels is reduced by a fraction 0·006 of the ground reaction, to account for rolling and mechanical friction[3].

The assumed profile drag breakdown of the aircraft shown in Fig. 14 is given in Table IV and speed and height variations are shown in Fig. 16. It will be appreciated that items " interference " and " leaks and gaps " can only be guessed at this stage. Control surfaces are likely to provide the bulk of the drag under the latter heading.

With the foregoing assumptions it can be deduced that in the flight path for maximum gain of height, some 15 seconds of maximum power are available after take-off during which time the aircraft reaches a height of 22 ft. and the speed drops from 55 to 50 ft./sec. Upon

TABLE IV

DRAG BREAKDOWN AT 50 FT./SEC. AND 12 FT. HEIGHT

	Drag area $ft.^2$	C_D *on wing area*	*Percentage of profile*	*of total*
Wing profile	1·31	0·0078	55	29
Fuselage below wing	0·27	0·0016	11	6
rest (fin)	0·12	0·0007	5	3
Tailplane profile	0·11	0·0007	5	2
Spinner	0·12	0·0007	5	3
Wheels	0·04	0·0002	2	1
Gaps, interference, etc.	0·40	0·0024	17	9
Total profile	2·37	0·0141		53
Induced drag of wing	1·99	0·0118		44
Induced drag of tail-plane	0·13	0·0008		3
TOTAL	4·49	0·0267		

reduction of power, still further increase of height may be had (say about 5 ft.) at expense of speed. Or, the aircraft can glide downwards reaching ground level after 25 seconds, when its speed is 55 ft./sec.; further gradual deceleration down to 45 ft./sec. would occupy another 40 sec. giving a flight time of 80 sec. The greatest duration possible—by flying all the time just above the ground—is no more than about 90 sec., reducing more or less linearly with height to 10 sec. above 22 ft.

8. Power Storage

The flight duration and ceiling of the aircraft are so limited that some assistance by storage of power before take-off would be valuable although its use in the first attempts is regarded as detracting from the impact of the initial achievement. However, subsequently its use would be unobjectionable, and indeed a great advantage. Power could be stored at a relatively low and non-fatiguing level of input before take-off, and released optionally (as an overhead drive) through suitable gearing at whatever torque power is required. Of the suggestions examined, the use of compressed air appears to introduce unacceptable losses in the efficiency of the conversion process, and the most promising idea (the subject of a patent application by W. G. Holloway, College of Aeronautics) involves the use of rubber stretched in a helix on a cylindrical drum (Fig. 17). One end is rotated by the input to stretch the rubber and the other, when released, provides the output torque. Frictional losses are minimised by constructing the drum surface as a series of rollers. Examination of the mechanics shows that it is best to use heavy gauge vulcanised rubber thread of high elasticity[12] (with a maximum elongation of something like 900 per cent). Power sufficient for three minutes of flight can be supplied by two starts of unextended length 15 ft. each, and 0·4 in. square cross-section, giving 45 windings round a one foot diameter drum (18 in. long); the maximum radial compression on full extension would be 420 lb./in.^{2} and the maximum gear ratio (on the output side) would be 18:1. Greater durations would require shorter, thicker lengths of rubber, necessitating higher gear ratios on both input and output sides, and generating higher compressions on the drum. A proper weight assessment of the device has not been made but it would seem likely to be about 10 lb./min. of flight duration. At least 150 ft. lb. of the stored energy is needed to compensate for the extra power required to carry each one lb. of the storage device's own weight.

9. Lateral Stability

The estimation of the stick fixed lateral derivatives has been aided by tunnel tests to determine the contribution of the isolated fuselage fin to the values of y_v, l_v and n_v (Fig. 19). The only remarkable feature of

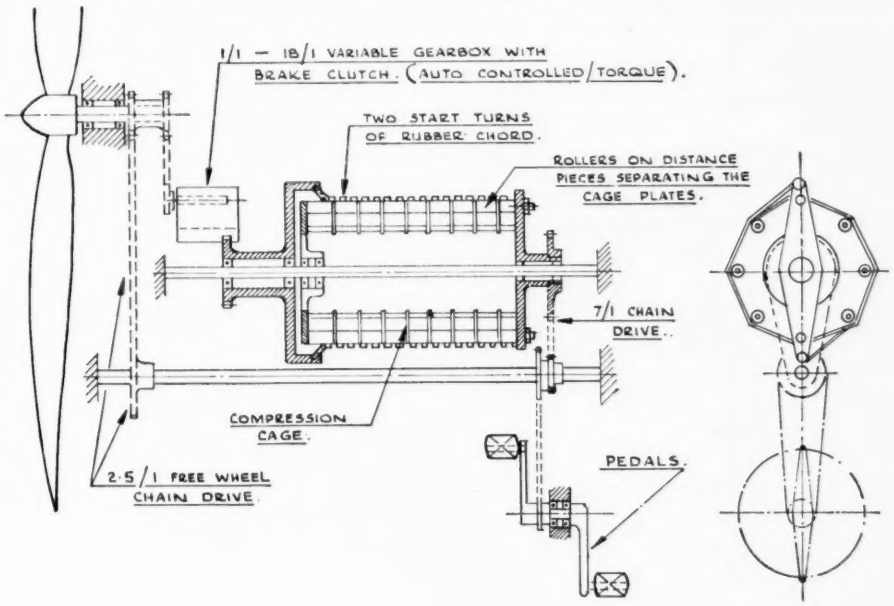

1/1 — 18/1 VARIABLE GEARBOX WITH BRAKE CLUTCH. (AUTO CONTROLLED/TORQUE).
TWO START TURNS OF RUBBER CHORD.
ROLLERS ON DISTANCE PIECES SEPARATING THE CAGE PLATES.
7/1 CHAIN DRIVE.
COMPRESSION CAGE.
PEDALS.
2·5/1 FREE WHEEL CHAIN DRIVE.

FIGURE 17. Helical rubber spring power storage unit. (*Patent applied for.*)

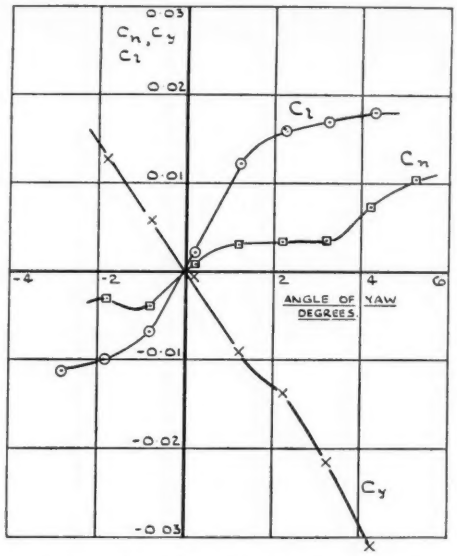


FIGURE 18. Tunnel results of lateral forces and moments on an inexact model of the fuselage-fin combination shown in Fig. 12. (Coefficient based on wing area and mean chord. Moments about c.g.)

these tests was the non-linearity of the rolling moment due to sideslip which increases suddenly in the destabilising sense up to 1° of sideslip, and then steadies off. (Thus the l_v contribution is something like 0·04 at zero sideslip — corresponding to $2\frac{1}{2}$° of wing anhedral—and then tends rapidly to zero). This behaviour is not at present understood, but for the purposes of these estimates the destabilising value of l_v from the body is taken to be cancelled by the favourable effect of wing-fuselage interference. This leaves the contribution to the wing dihedral alone, which it will subsequently be shown ought to be large, and so we take it as 5°.

No oscillation tests on the fuselage have yet been made: its contributions are guessed as $y_r = 0{\cdot}04$, $n_r = -0{\cdot}015$. Table V gives a list of all derivatives relevant to gliding flight at 50 ft./sec.

The modes of disturbed motion of the aircraft as drawn, and with the assumed c.g. position, have a component which is unstable, and which can be identified as a very slow spiral divergence (doubling in 20 sec.). This is due to the rear equality of the terms $l_v n_r$ and $n_v l_r$, the latter being required to be less than the former for stability. An instability of this mode would not be unusual on aircraft operating at high C_L (due to large l_r), but in view of the special nature of the projected aircraft it would seem best to try to ensure stability. As much more increase in l_v (*i.e.* wing dihedral) is unreasonable, the easiest remedy is to decrease n_v which may be done by extending the fuselage forward. It is estimated that an extension of 2 ft. would move the c.p. of the lateral forces by half this distance and decrease n_v by 0·007 to half its present value.

If this is done the spiral motion is virtually in neutral stability. This should be adequate and the few response calculations attempted do not suggest any troubles resulting even from the choice of zero n_v. The other modes of motion are virtually unaffected by this measure, and consist of the very rapidly damped subsidence in roll (half amplitude time 0·04 seconds) and a "weathercock" motion damped to half amplitude in 0·7 sec. but with so long a period (8 sec.) that its oscillatory nature would not be noticed.

It is only when the recommended decrease in n_v is accompanied by a decrease in dihedral that the lateral motion becomes adversely affected, because of the closing-up of the stability boundaries (Fig. 19). For this reason it would seem best to preserve the present large dihedral, even if the destabilising l_v of the fuselage is a fiction. The stick-free stability has not yet been studied but the spiral damping could easily be improved in this condition (if considered necessary) by allowing the rudder to trail in sideslip[13]—so reducing n_v still further. It should be noted that the foregoing estimates (as well as those of the next paragraph) have included only a token allowance for propeller-fin effect based on a guessed value of $y_v = -0{\cdot}02$. Its effect on n_v (and the spiral motion) is relatively important, but a more accurate assessment could only be made when its design is fixed.

10. Longitudinal Stability

The estimated stick-fixed logitudinal stability derivatives of the layout given in Fig. 14 are quoted in Table VI relative to flight at 6 ft. height and $C_L = 0{\cdot}94$ (corresponding to a speed of 50 ft./sec.). The aircraft has been designed to provide a static margin of 0·05c without propeller-fin effect (and 0·08c with). The value of m_u is zero for gliding flight, and on the debatable assumption that constant power is maintained in any disturbed motion, equals 0·009 in the cruise condition, and 0·022 at full power (1.25 h.p.). If merely r.p.m. is kept constant, the value of m_u would be even higher. This increase in m_u unfortunately heightens an instability in the phugoid motion—increase in speed caused by a dive gives rise to a drop in thrust, and a nose-up

TABLE V

ESTIMATED LATERAL DERIVATIVES

$y_v = -0{\cdot}235$	$y_r = 0{\cdot}014$	$y_p = -0{\cdot}003$
$n_v = 0{\cdot}014$	$n_r = -0{\cdot}022$	$n_p = -0{\cdot}09$
$l_v = -0{\cdot}087$	$l_r = 0{\cdot}21$	$l_p = -0{\cdot}56$
$i_A = 0{\cdot}030$	$i_C = 0{\cdot}038$ $i_E = 0{\cdot}0052$	$\mu_2 = 1{\cdot}21$

(Notation as in Ref. 14)

TABLE VI

ESTIMATED LONGITUDINAL DERIVATIVES

$x_u = -0{\cdot}032$	$x_w = 0{\cdot}41$	$x_q = 0{\cdot}020$
$z_u = -0{\cdot}94$	$z_w = -2{\cdot}67$	$z_q = -0{\cdot}45$
$m_u = var.$	$m_w = -0{\cdot}059$	$m_q = -0{\cdot}16$
$i_B = 0{\cdot}109$	$\mu_1 = 3{\cdot}66$	$m_{\dot{w}} = -0{\cdot}02$

(Notation as in Ref. 14)

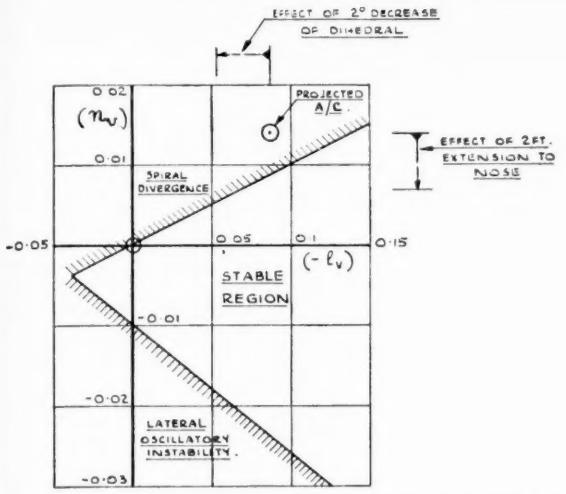


FIGURE 19. Lateral stability diagram for projected aircraft. (Illustrating the need for forward extension of the fuselage.)

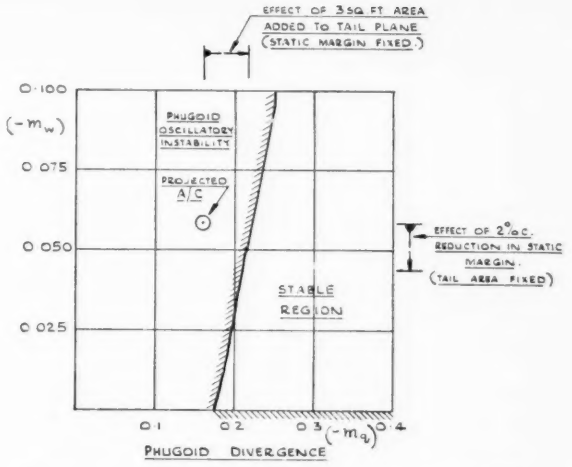


FIGURE 20. Longitudinal stability diagram for projected aircraft. (Illustrating need for more tail area, moving wing forward to preserve the same static margin.)

pitching moment is provoked (due to the offset thrust line) which over-corrects and amplifies the oscillation.

Even with power off—in the glide condition—the phugoid has a very slow divergence. At full power it diverges to double amplitude in 7 seconds with a period of the same time: an unpleasant motion, although it must be recalled that the duration at full power is no more than two periods. Apart from the contributory aggravation of the offset thrust line its root cause is the inadequate damping in pitch (Fig. 20). Tail arm cannot be increased without giving trouble in the lateral motion (since with the layout assumed this would mean increasing fin area and n_v), but an increase in tail area by 40 per cent or more (*i.e.* up to 12 sq. ft.), adjusting the wing position so as to keep the static margin unaltered (or perhaps decreasing it a little) would be quite acceptable, and would neutralise the instability even at full power, while in the glide the time to half amplitude would be 12 seconds, the period 4 seconds.

The rapid incidence adjustment is virtually independent of this effect or its remedy. It is well damped (to half amplitude in $\frac{1}{4}$ sec.) with relatively so long a period (3·5 sec.) as to be unnoticeable. As with the lateral motion, it is anticipated that the stick-free stability would be easy to guarantee.

11. Control

It is evident that—quite apart from its effect on the phugoid—the off-set thrust line is going to give some awkward longitudinal control problems. Simulator tests would be most valuable in determining just how much difficulty there would be. If it were decided that it forms too great an embarrassment, and it is admitted that the propeller cannot be much lowered, it is not inconceivable that some simple kind of servo control to the elevator, activated by a torque meter applied to the propeller shaft, could be evolved.

The problems under this present heading are those most needing attention on the part of the aerodynamicist. There is the general problem of what movement of the "handlebar" could be used for elevator control, since the cyclist pulls on his arms during strenuous action. There is the problem of size and shape of control surfaces, compatable with the likely restricted aerobatic qualities of the under-powered and fragile machine, and the need for low-drag: for instance, perhaps wing warping is preferable to the provision of ailerons despite the low rolling power implied.

12. Conclusions

(*i*) It is recommended that the cycling action be used on the grounds that men who are well practised in it, and the powers they can generate, are well-known. Hand-cranking by the second crew member would seem to give some extra advantage.

(*ii*) With the two cyclists capable of power outputs close to National record levels the performance of a projected fixed wing aircraft is assessed and shows that it can be taken off from the ground, flown for durations of up to 1½ minutes, and reach altitudes of up to 25 ft. by the sole use of muscular power.

(*iii*) For flight to maximum height a high rate of power output is required at the end of the take-off run and during the climb, while a low "cruise" level suffices at other times. For level, long-duration flights, intermediate and constant power output is most effective.

(*iv*) Considerable improvement of performance will result from the use of energy stored before take-off. A device using rubber as the energy storage medium is suggested.

(*v*) A wing section (the NACA 65A(10)12) has been tested at the flight Reynolds number, and has about as much camber as it would appear desirable to employ.

(*vi*) A high wing monoplane design has been considered and is shown in Figs. 12 and 14. Its novel features are a fin-shaped fuselage, single track wheels directly driven for take-off, and a high position for the pusher propeller. The optimum wing loading has been found as about 3 lb./ft.^{2} and the best span as about 60 ft.

(*vii*) Examination shows that the projected aircraft needs more tail area to ensure longitudinal stability, and an extension to the nose of the fuselage to provide lateral stability. The shape of the wing plan also needs detailed change.

(*viii*) A wing covering of $\frac{1}{32}$ in. thick ply is the best so far considered if its surface is to be rigid, and wing weights of about 0·45 lb./ft.^{2} have been calculated.

(*ix*) A number of specific suggestions for future work emerge. These include:—

- (*a*) A reassessment of the optimum wing geometry in view of the facts elicited by the present study.
- (*b*) Tunnel tests on practical construction, rigid and fabric covered, wing sections; on the lateral characteristics of a modified fuselage shape; and on the flow field at the wing-body junction.
- (*c*) Consideration of the design of a control system and control surfaces.
- (*d*) Simulator tests to determine the handling qualities, backed by response calculations.
- (*e*) The design of a suitable propeller.
- (*f*) A relevant theoretical study of ground effect.

(*x*) Longer term research work might be devoted to:—

- (*a*) An assessment of the advantages of helicopter, or other aircraft form.
- (*b*) An assessment of other methods of power generation.
- (*c*) The design of the power storage unit.
- (*d*) The development of wing suction.

Acknowledgments

The author would like to thank his past colleagues of the College of Aeronautics, and many other enthusiasts, for their help and stimulating criticism which find expression in this paper. In particular a special acknowledgment is due to B. S. Shenstone of B.E.A., who was primarily responsible for initiating interest in this remarkable problem.

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Turbo-Prop Testing

Evolution of the T-Bench for Dart Overhaul Bases

by

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1. Introduction

A number of airlines throughout the world will, in the near future, be reconditioning and testing Dart engines from Viscount, Friendship and Herald aircraft. In most cases the testing will employ a technique which has been in use at the Rolls-Royce Australian overhaul base at Sydney, N.S.W., since August 1954. This is based on the torque reaction of the engine to the propeller and is suitable for testing any comparable engine.

The history of propeller-turbine testing at Rolls-Royce is given, with the reasons underlying the introduction of torque reaction testing and the experience and information accumulated in 2,250 hours running at Mascot is listed, starting with unique equipment and inexperienced personnel. This Australian experience has resulted in the "T"-bench test stand, which was designed and developed by John Curran Ltd. of Cardiff as a private venture (British Patent No. 745211), becoming the recommended method of overhaul test for the Rolls-Royce Dart engine (*see* Figs. 1 and 2).

2. Dart Acceptance Test Requirements

The piston engine is accepted on its power at a fixed r.p.m. and boost (manifold pressure). Experience has proved that, while the maximum take-off r.p.m. of the Dart is fixed, it is impossible to declare a given J.P.T. (jet pipe temperature) as the value at which acceptance power is obtained. This apparent lack of consistency between engines is better understood when it is realised that, of the power developed at the turbine, only approximately one-third is measured at the propeller shaft, the other two-thirds being absorbed by the compressor and various auxiliary drives. Roughly speaking if there is a 2 per cent drop in turbine and compressor efficiencies, this will result in a 10 per cent loss of shaft horse power. Considering a similar case with a highly supercharged piston engine, 2 per cent loss of efficiency will only result in about 3 per cent drop in shaft horse power. It will be seen that, whereas the nominal minus 4 per cent piston engine power limitation is a practical standard of acceptance, a tremendous number of test rejections are going to occur on the propeller-turbine if a given r.p.m. and J.P.T. are to be mandatory for acceptance power conditions.

Accordingly the requirements of Dart test are such that we have to be able to assess the J.P.T. at which each individual engine produces its nominal power. As an easement we are allowed to accept turbo-prop engines in the power bracket—nominal plus or minus 4 per cent. Power curves must therefore be run on each engine and from these curves the J.P.T. limitations are picked off to give acceptable power.

To obtain the necessary figures the Dart must be run at a controlled r.p.m. which varies from the nominal r.p.m. as a function of the absolute A.I.T (air intake temperature). While controlling at the appropriate r.p.m., we must be capable of varying and recording the torque. Inconsistencies in the engine torque recording system preclude using it for measuring power output and accordingly some external means of doing this is a necessity.

All the normal requirements of engine test such as starting, acceleration, water-methanol setting, oil consumption measurement and temperature control, fuel metering and interconnection, must also be met.

3. Test Equipment in Use at Rolls-Royce

All early Dart development work was done on the normal water dynamometer which has been the test medium for tens of thousands of piston engines from

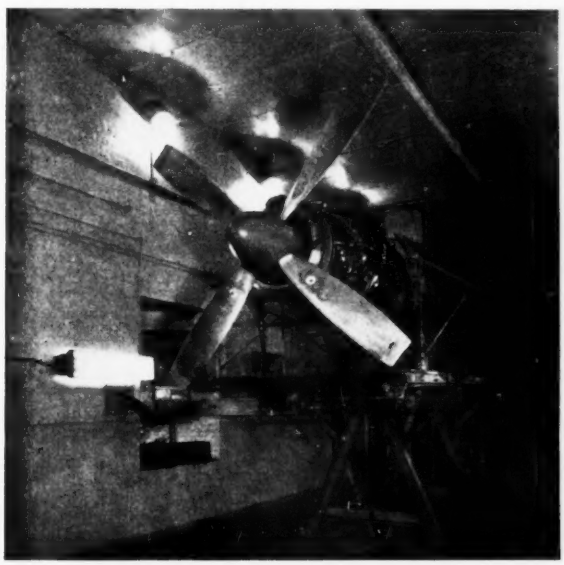


FIGURE 1. T-bench test cell at Mascot.

Originally received 23*rd August* 1957—*Revised March* 1958.

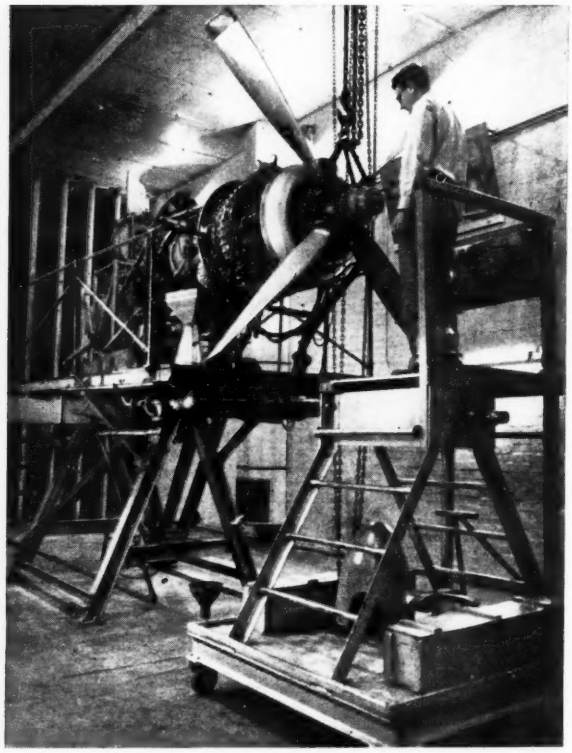


FIGURE 2. Mk. I T-bench (Propeller storage and mounting stand shown).

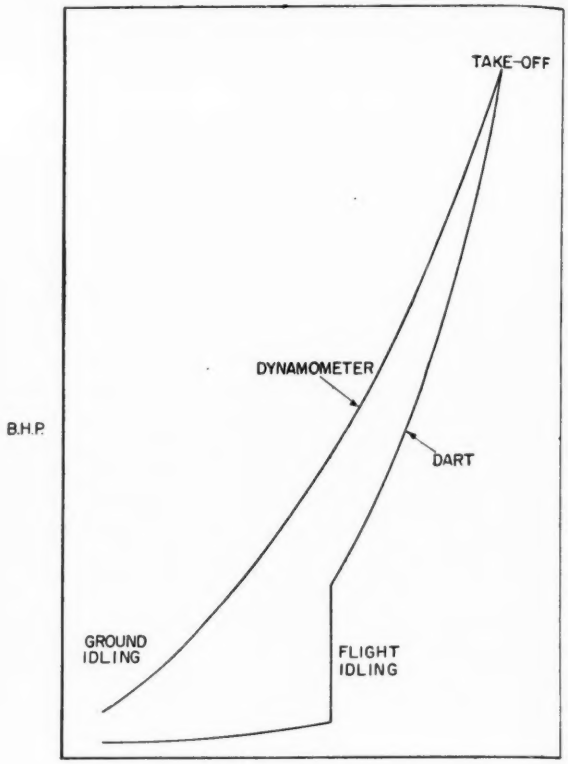


FIGURE 3. Comparison of Dart power curve with water brake characteristic.

the days of the First World War right up to the 2,000 plus b.h.p. engines of the early forties.

The water brake has three unsatisfactory characteristics when it comes to testing the Dart: —

(*a*) The shape of the power absorption against r.p.m. curve.

(*b*) Inability to make almost instantaneous torque variations.

(*c*) High starting torque.

Figure 3 shows a curve of maximum power output limited by J.P.T. at varying r.p.m. for the Dart, superimposed on a water brake characteristic.

It will be seen that, if the engine is running at maximum power conditions, *i.e.* full throttle at maximum r.p.m., closing the throttle will result in the overheating of the engine, as the brake load at the lower r.p.m. is in excess of the maximum torque that the engine can supply without over-heating. This is obviously undesirable and can well result in severe damage to an engine merely by the operator obeying what instinct and training has taught him to do (*i.e.* throttle back) in the event of any unforeseen occurrence during test.

Tied up with this is the slow rate of sluice (torque) adjustment, even with motor actuation.

The piston engine analogy of excessive J.P.T.—excessive boost—will result in detonation. This in itself is not critical over a period of seconds and normal adjustment of the brake is quite satisfactory. However, with excessive J.P.T., five seconds can mean the difference between safety and disaster to the "hot end," and the relatively sluggish adjustment of the sluices is far too slow to keep the engine out of danger.

The Dart is incapable of starting without overheating if there is more than a minimum loading on the propeller shaft. This has necessitated the introduction of flight propellers having an ultra-fine pitch for starting and ground running. It is obvious that the friction in the dynamometer glands (which are necessary to prevent water leakage), together with the high inertia of the line shaft and rotor, can bring about a state of affairs where normal starting conditions are unobtainable.

It will be apparent from this that the water brake is generally unsuitable for Dart testing.

During the Second World War vast strides were made in the development of the electrical or dynamatic dynamometer and the water brakes at Rolls-Royce were gradually replaced by this new form of power absorption. It was therefore logical, when the Dart found a considerable market, that the testing of the engine be transferred to the new type dynamometer which was capable of constant speed control. In other words the controls of the dynamometer can be set to the equivalent of a given engine speed while the engine is still running at idling conditions and opening the throttle will increase the revs. at minimum torque until the selected r.p.m. is reached; further opening of the throttle will cause the

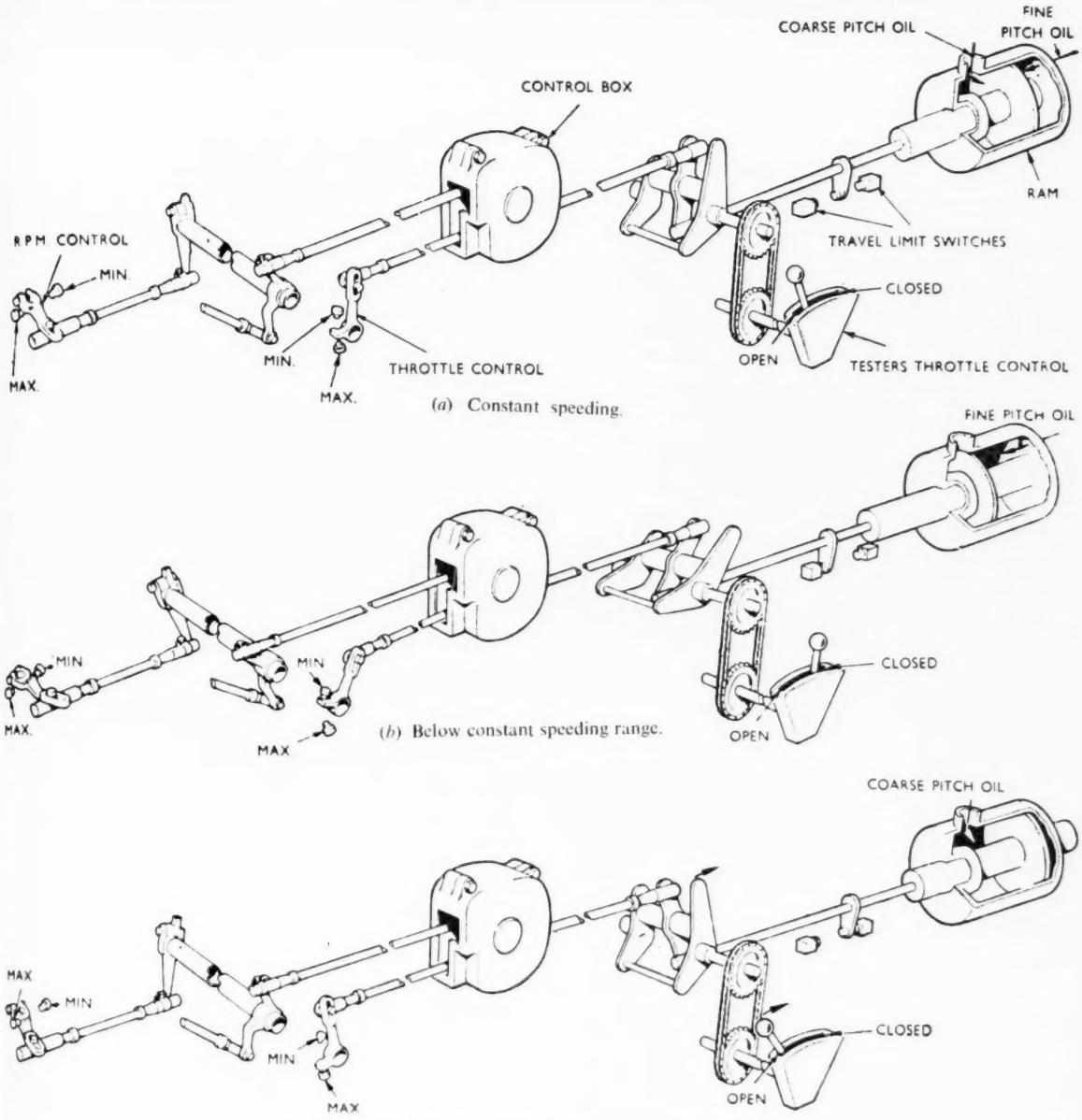


FIGURE 4. Dart control interconnection—simulator for use on dynamatic dynamometer.

load to build up at this r.p.m. as it does with a constant speed propeller.

Conversely, if the throttle is shut from maximum power conditions the load comes off the dynamometer and the tendency to over-heat is no longer present. Initially some difficulty was encountered due to lag in the electrical controlling system of the dynamometer. On accelerating, the revs. would build up considerably above the selected r.p.m. before the brake torque was increased. Further development work resulted in a system with very little response lag and over-shooting of r.p.m. was reduced to a minimum. Torque changes are now isochronous with control adjustment.

The dynamatic dynamometer still has the disadvantage of a very high inertia in the rotors, this being even more pronounced than on the water brake, but in general it is a suitable device for Dart testing.

Early engines were rated on the dynamometer and then passed to the test hangar where they were run with a flight propeller to interconnect the controls. Interconnection is basically linking of the throttle valve to the r.p.m. control lever; adjustments to control geometry ensure that, at any given r.p.m., the fuel flow and accordingly the jet pipe temperature are within acceptable limits.

With the engine r.p.m. controlled purely at the

discretion of the tester, there seemed no method of interconnecting on the test bed. However, as the time entailed by hangar control adjustment was considerable and was tending to create a bottle-neck, an ingenious method was developed for use on the dynamometer. Basically the oil pressure relationship, fine to coarse, was determined for propeller constant speeding. A hydraulic ram was constructed to control under these conditions and was put into the throttle/r.p.m. control linkage via a differential (Fig. 4). Using this method the tester sets the dynamometer and accordingly engine r.p.m. and the relevant throttle position is selected by the ram. Interconnection over the appropriate range is then progressively checked in the normal manner.

4. Factors Necessitating Alternative Test Equipment

Following the success of the Viscounts in B.E.A., it became apparent that considerable numbers of these aircraft were going to be sold throughout the World. In contrast to European operators, it was not going to be a practical proposition for many overseas airlines to return their engines to the parent factory for overhaul.

The re-tooling of existing aero-engine overhaul plants to undertake this work obviously presented certain difficulties, but the greatest problem was that of testing the engine.

A large throughput of engines can justify the purchase and installation of test equipment of great capital value and requiring highly skilled maintenance personnel. An outlay of this magnitude is economically unsound from the point of view of the small operator, who will possibly be overhauling less than one engine a week. An investigation of alternative methods of testing the Dart was therefore put in hand with a view to introducing some relatively inexpensive means of carrying out acceptance test requirements.

5. Consideration of Alternative Test Methods

5.1. THE CALIBRATED CLUB

In the past the most economical method of aero-engine testing has been the use of the calibrated club, which at a given r.p.m. will absorb a certain power varying only with ambient conditions, for which corrections can readily be applied. For a piston engine where the power variations are such that full throttle conditions will give an acceptable degree of under- or over-speed, this method is satisfactory.

However, with the wide variations of power that are experienced on the Dart, under-speeding (which results in a drop in the air/fuel ratio at full throttle conditions) becomes excessive. This is accentuated by the high power loss with increase of A.I.T. (approaching one per cent per degree Centigrade) which is inherent in the engine.

We are further limited to a 1·725 per cent over-speed for test purposes, as compared to 5 per cent or more with the piston engine.

To cover acceptable power variations over an I.S.A. $\pm 20^{\circ}$C range of ambient temperature would necessitate having at least five test fans in approximately 1° steps of pitch. This in itself is not very satisfactory. Furthermore, there is no means of knowing which of the test fans is appropriate. Having fitted the wrong one and by experience assessed which fan is needed to absorb the power within acceptable limitations, the ambient temperature and/or wind velocity can alter sufficiently to make this second fan unsuitable.

The calibrated club therefore is not a practical proposition for Dart test.

5.2. THE CALIBRATED VARIABLE PITCH FAN

In view of the drawbacks to the conventional calibrated club, the question of calibrating a variable pitch propeller throughout its pitch range and of incorporating mechanism to indicate the actual pitch at which the propeller was constant speeding was investigated. If some means of recording the pitch under running conditions could be devised and the appropriate power absorption of the propeller determined from calibration curves, a satisfactory means of testing the engine would be available.

Unfortunately, no means of doing this could be found, the degree of accuracy of pitch measurement required (at least within 0·1°) being far in excess of the potential sensitivity of any indicating mechanism.

Another possible source of error was in the assumption that all blades run at a consistent pitch relative to the master (indicating) blade. Variations are possible due to backlash in the blade controlling mechanism, and this in itself could result in discrepancies of 50 b.h.p. at maximum r.p.m.

5.3. CALIBRATED CENTRIFUGAL COMPRESSOR

The possibility of using the compressor of a Derwent engine as a means of power absorption seemed to hold considerable promise.

The complications of a gearbox to step up the Dart propeller shaft r.p.m. with the associated problems of lubrication and heat dissipation were not insurmountable and the compiling of power curves from the compressor by means of throttling was quite feasible, provided that the initial calibration was accurate and that the compressor suffered no deterioration in efficiency.

However, interconnection still remained a problem, and it was proposed to do the necessary testing when the engine was installed in the aircraft. This obviously had considerable drawbacks.

5.4. THE TORQUE REACTION HANGAR

One of the theoretical methods of checking power on aero-engines was the torque reaction hangar. This was employed by the Germans in the First World War and, with the degree of accuracy that was considered necessary in those early days, had some success. Little progress was made in torque reaction testing due to the limitations of the equipment and the introduction of satisfactory water brakes.

About fifteen years ago, two manufacturing companies in the United Kingdom installed torque recording mechanism in their test hangars, but in spite of considerable effort these units never proved satisfactory, the guaranteed order of accuracy being something like

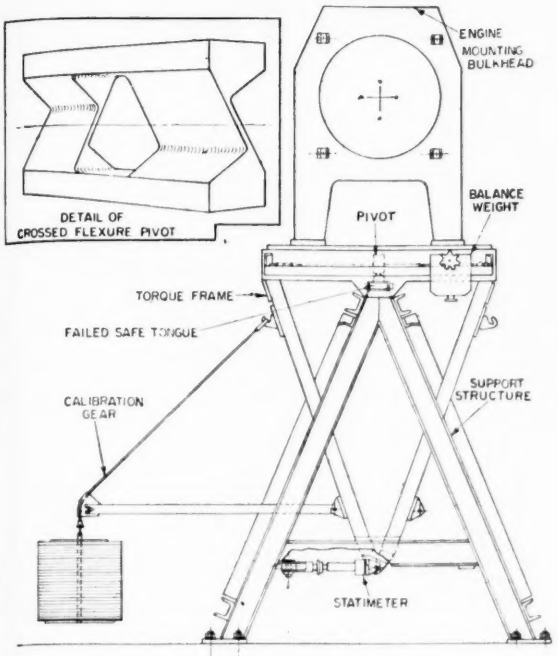


FIGURE 5(*a*). Front elevation of T-bench.

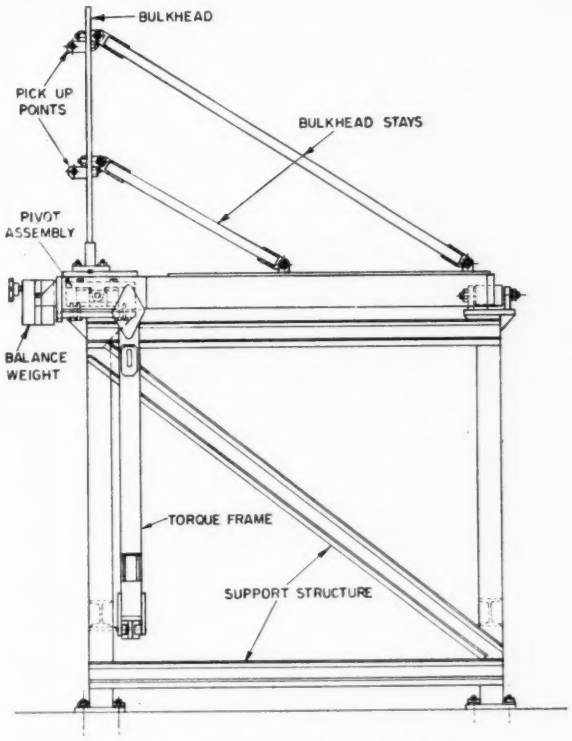


FIGURE 5(*b*). Side elevation of T-bench.

150 b.h.p. The inaccuracies were mainly due to friction and hysteresis in the pivoting assemblies which were designed to allow the engine rotation about the propeller shaft axis.

However, as a torque reaction hangar could, theoretically, carry out the requirements of Dart test, using the pitch controller and propeller as a load absorbing and r.p.m. controlling medium, it was decided to make a thorough investigation of the fundamentals of this type of test bench.

A study of all possible suspensions allowing the engine freedom to rotate about the propeller shaft axis failed to reveal any method that was not likely to suffer from friction. It began to look as if the torque reaction hangar was doomed to remain an attractive possibility limited by its practical defects.

At this point, however, consideration was given to the idea of mounting the engine to pivot about a line parallel to the propeller shaft axis. A study of the mechanical principles involved revealed that theoretically there was no reason why such a form of mounting should not be used and further, that the distance between the pivot line and the propeller shaft axis was completely immaterial from the point of view of torque measurement at a given radius from the pivot.

This new theory and the limited arc through which the assembly was to rotate allowed the cross flexure pivot to be introduced into the design. At this stage (November 1952) John Curran Ltd. of Cardiff, the Air Ministry Test Equipment Contractors, were approached with a view to their manufacturing a bench based on this new principle.

6. T-Bench General Design Specifications

Considerations of aerodynamic cleanliness, scope for future development and ease of mounting the engine, led to the adoption of a design which had the platform and bulk-head sitting on the cross flexure links with the torque restraining arm mounted underneath (*see* Figs. 5(*a*) and (*b*)). A "failed safe" device was incorporated to take care of any possible failure of the pivots.

Bearing in mind that the qualities of reliability, accuracy and economy were mandatory, wherever possible specifications were for equipment which did not require special parts that might be unobtainable in the locality of the test set up, and which could be repaired by a fitter as distinct from a specialist. Refinements were left to be incorporated at a later date.

To avoid delays in material deliveries, mainly channel, as distinct from tubular, construction was used. Lugs to accommodate a Jurystrut, with cable and weight pan attachment, were built on to the side of the torque restraining V-frame for calibration purposes. The stand was stressed for 2,500 h.p. and the effective length of the torque restraining arm was made 63·025 in. so that a b.h.p. constant of 1,000 would be applicable.

$$\left(\frac{\text{torque} \times \text{prop. r.p.m.}}{1{,}000} = \text{b.h.p.}\right)$$

The calibration strut was made of equal length to get a 1 : 1 ratio for dead weight test purposes.

A statimeter was considered to be the most compact and readily incorporated means of measuring torque and was mounted in a horizontal strut between the

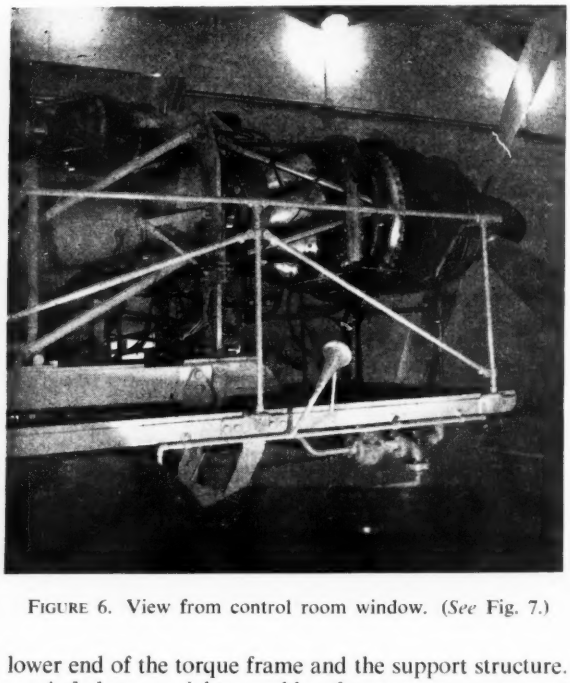


FIGURE 6. View from control room window. (*See* Fig. 7.)

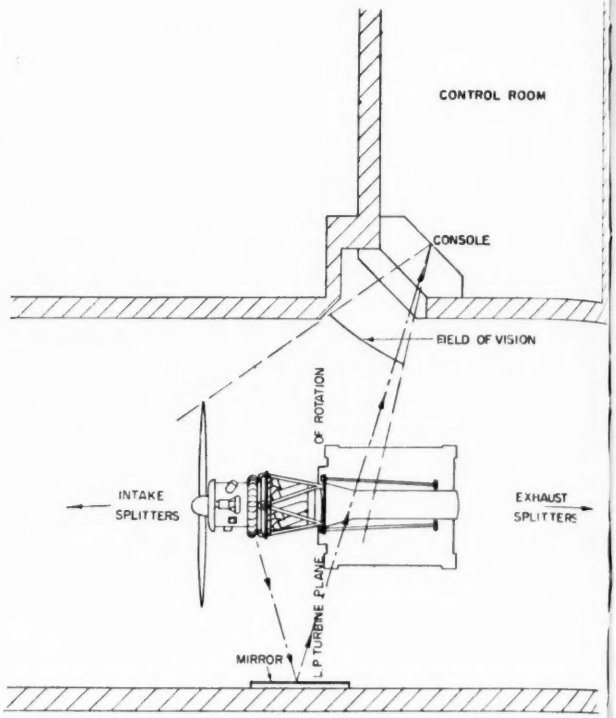


FIGURE 7. Plan view of engine and control room layout.

lower end of the torque frame and the support structure.

A balance weight capable of transverse movement was fitted to the platform so that the centre of gravity of the engine/platform assembly could be adjusted into the vertical plane through the pivot; this eliminated false torque indications from this source.

All controls were arranged so that they exerted the minimum out-of-balance loads about the pivot axis. Wiring, control runs, and so on, were built into a duct, which attached to the test stand at one end and to the instrument console at the other. All electrical equipment was, as far as possible, built into a box beneath the control console.

For ease of engine installation and changing of thermocouples during test, the jet pipe was made to be retractable and was held in position by spring-loaded plungers.

To reduce spares holdings the fact that the operator of the T-bench would have access to Viscount parts was taken into consideration. The engine starter panel, feathering relays, mounting strut, auxiliary gearbox drive and r.p.m. generator were standard aircraft parts. The aircraft type r.p.m. indicator was considered to be too small for test purposes, when a 10 r.p.m. margin is the maximum that can be allowed. Accordingly a 3 in. dial 1,000 r.p.m. per revolution indicator was fitted and provision made in the wiring for plugging in a tuning fork actuated stroboscope.

The console instrument layout was designed so that all significant gauges were suitably grouped and could be seen with the minimum of eye movement. All manually-operated controls were put within easy reach of the operator, who was ensured a full view of the engine while being safely placed in the event of failure of any of the rotating assemblies (*see* Figs. 6 and 7).

The design incorporated all safety features which had been developed over years of experience at Rolls-Royce. These included a five-minute initial starting delay in the electrical and fuel supplies to the T-bench. During this period all places where ignitable vapours could have accumulated are thoroughly purged with fresh air. Any items such as roller shutters that need to be in a given position to ensure safe operation are electrically interlocked with the starting panel, and all fuel drains run through a water sealing tank to a collector tank. This latter feature eliminates the possibility of any fire in one location flashing back, not only to the fuel dump tank, but to another location via the drains.

7. Ancilliary Equipment

7.1. PROPELLER

The nature of the Curran T-bench rendered any form of calibrated test fan unnecessary. The equipment specified was normal flight propeller and spinner, less the de-icing gear.

A study was made of all the factors entailed by running the engine behind the propeller as distinct from in still air, as on the dynamometer. This was to ensure a better understanding of any variations in performance due to ram, slipstream helix, and so on, that might be apparent when calibration runs were made.

7.2. HANGAR BUILDING

To avoid false torque indications it is essential for the air to be free from turbulence and to enter the propeller axially; accordingly a test cell was designed in conjunction with the test stand.

On the assumption that the ultimate power of the Dart would not be in excess of 2,500 h.p., calculations indicated that a 19 ft. square section would be large enough to cope with the maximum air flow. The T-bench, as it was now becoming known, was therefore designed so that the propeller shaft of an installed Dart was 9 ft. 6 in. from ground level, *i.e.* on the centre axis of the proposed test house.

Vertical splitters to serve the double function of air straightening and silencing were designed by Detuners Ltd. (London). Tests indicated that mixing of the jet efflux with the propeller slipstream reduces the mean temperature to such a level that no form of exhaust duct is necessary. The exhaust and intake splitters were therefore made identical except that the former had metal instead of wooden frames.

7.3. OIL SYSTEM

Maintenance of engine oil temperature at scheduled requirements necessitates both heating and cooling of the oil. No attempt was made to use the engine cooler for this function due to the difficulty of designing equipment which could be quickly and conveniently attached during the mounting of the engine.

The slave system was incorporated on the test stand itself, and consisted of a water cooler and electric immersion heater in parallel with a splitter valve controlled from the instrument console. Connections to and from the system were designed to mate with corresponding engine parts; all flexible pipes were made of synthetic rubber to preclude attack by the DERD.2487 lubricant.

7.4. AUXILIARY LOADING

Since air was to be the main power absorbing medium, consideration was given to the possibility of dispensing with a water brake to absorb the 75 b.h.p. from the auxiliary gearbox drive. A small eight-bladed fan with variable shrouding was found to be capable of absorbing this power (*see* Fig. 8).

The fan had the advantages of dynamic balance and an almost indefinite overhaul life. The lack of any services obviously made the installation far cleaner in all respects and such troubles as freezing in sub-zero temperatures were eliminated.

7.5. DE-MINERALISATION PLANT

Until 1953 distillation had been the only approved method of producing water of the purity required for the Dart water-methanol mixture. In view of the expense of distilled water, steps were taken to get de-mineralised water accepted as an alternative.

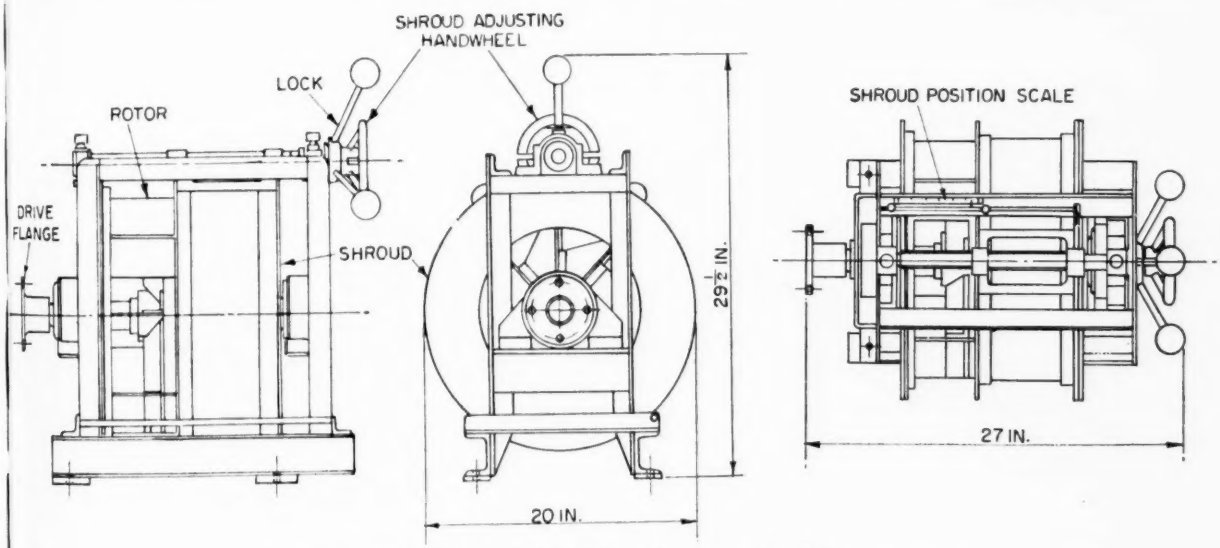
As a result of this clearance a de-mineralisation plant capable of producing forty gallons of pure water per hour was included in the specification.

8. Initial Calibration

Five months after the conception of the T-bench, John Curran Ltd. completed a prototype stand and initial calibration running was begun. After a number of minor snags had been rectified it was apparent that consistent power curves could be produced.

Unfortunately the available test house was of a very poor aerodynamic design and strain gauge tests on a flight sized Dart 505 propeller, running in the resultant turbulent air, revealed excessive blade root stresses. In view of the novel nature of the equipment and as insurance against possible trouble, the propeller blades were cropped by 10 per cent, when the stress level became satisfactory. As expected, cross checks revealed that cropping showed no appreciable effect on the power curves. A total of approximately 12 hours running time was all that was necessary for this work.

The next stage was the construction of the first production model which was shipped to Australia early in 1954. This test stand was installed by locally recruited



Shroud shown in the full open (maximum torque) position.
This dynamometer absorbs 75 b.h.p. at 5,000 r.p.m. and approximately 3 in. shroud opening.
FIGURE 8. Auxiliary drive dynamometer (Walker Type L fan).

personnel. Of these, one had a very limited previous experience of pure jet testing and none had any experience of hangar testing.

During the installation work all systems were thoroughly checked and every precaution taken to eliminate possible sources of trouble at a later date. In particular, all fuel pumps in the ring main system were left circulating for 24 hours, and the feed to the Dart fuel panel was purged full bore from a 2 in. hose to ensure the maximum scavenging effect.

As a result of these precautions, a first time light-up was obtained when the initial engine was installed and no troubles due to dirt in the fuel or items of that nature have occurred since.

Preliminary running was done in August 1954 and was followed immediately by complete calibration checks of a far more comprehensive nature than those undertaken in the U.K. These checks confirmed several anticipated factors: —

(*i*) Power at a given r.p.m. and fuel flow agreed with dynamometer figures, but the indication on the statimeter was approximately 2 per cent low as a result of slipstream helix effect on the engine and stand.

(*ii*) Compressor delivery pressures were in excess of those recorded on the dynamometer, due to the ram effect behind the propeller.

(*iii*) J.P.T. at a given r.p.m. and fuel flow was, as a result of the higher air flow through the engine, approximately 10°C lower than dynamometer figures, but lined up with figures obtained with the engine in the aircraft.

We were anxious to retain the 1,000 factor in the b.h.p. estimations to avoid a further complication in what was already an involved calculation. To do this it was decided that we would attach a correcting fin to the pivoting platform. Calculations indicated that an angle of 45° to the fore and aft axis was the optimum from the point of view of insensitivity to change of helix angle. The final dimensions of the fin were established experimentally and were such that the power recorded on the statimeter lined up with dynamometer and engine torquemeter figures.

Subsequent experience has indicated that the identical fin is satisfactory for use on the Dart 510, in spite of the lower reduction gear ratio, higher b.h.p., and resultant slipstream variations.

It appeared that the effects of (*ii*) and (*iii*) would be to bring us into line with installed performance, where "Installed coolness," compared to dynamometer figures, was causing some concern. However, as engines tested on the Derby dynamometers were the yardstick for aircraft acceptance performance, T-bench engines are interconnected to temperatures 10°C higher than those established by the rating curves.

8.1. OIL SYSTEM

Experience indicated that at less than 15° A.I.T., the heater was incapable of maintaining the required oil inlet temperature. The capacity was therefore increased to 10 kW. Temperatures within these limitations were extremely easy to control at any given level. To eliminate inconsistencies of oil consumption readings, air bleeds were tied from the top points of both cooler and heaters to the oil return line to the engine cooler.

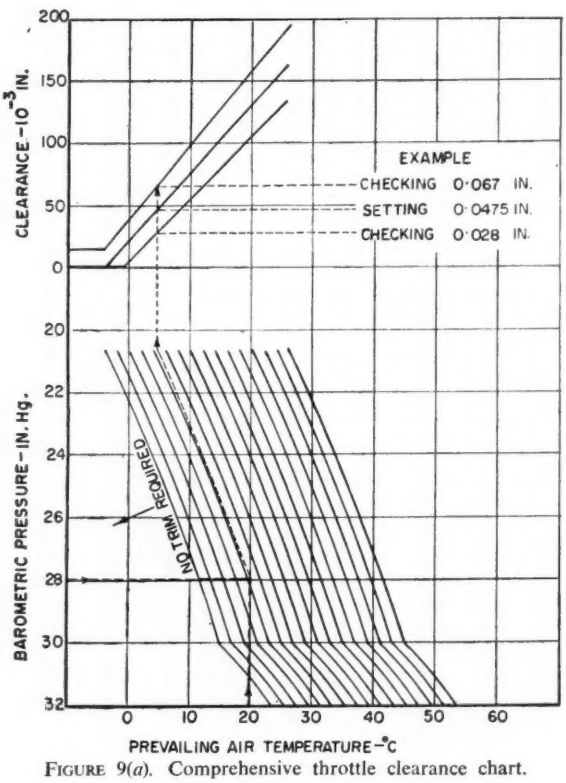


FIGURE 9(*a*). Comprehensive throttle clearance chart.

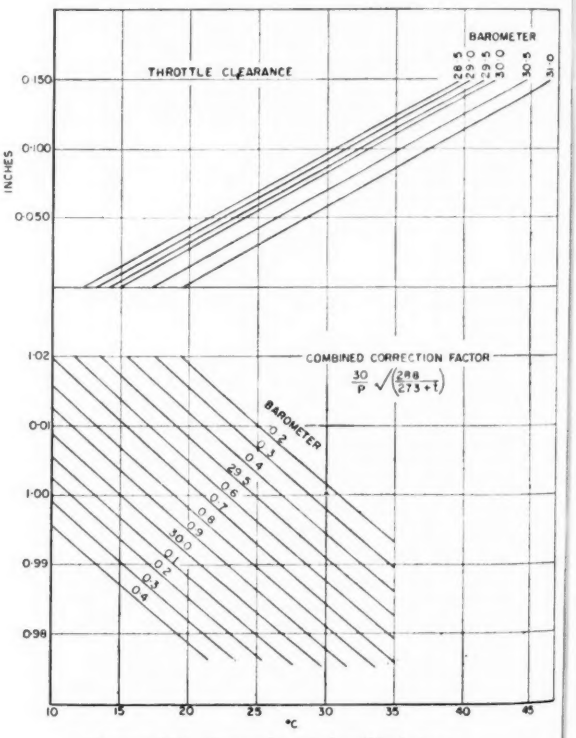


FIGURE 9(*b*). Examples of simplified charts.

9. Crew Training

Test crews were given a series of lectures on the basic principles of the gas turbine and on various features applicable to the Dart. The problems of engine handling and interconnection were given considerable emphasis. Great stress was, and continues to be, laid on the observation of the J.P.T. gauge and there have so far been no cases of over-heating or damage due to mal-handling.

A complete and accurate log of all starts and running is maintained and serves as a useful reference in the event of queries arising in service. In particular the maximum J.P.T. during the starting cycle is always recorded and the necessary observation of the gauge to carry out this requirement has several times resulted in engines being saved from even momentary over-heating.

During the initial false start and blow-out cycle, a tester is always stationed inside the cell and as a result of experience is able to detect any unusual note in the engine before any light-up takes place. The steady motoring r.p.m. during the blow-out cycle is also recorded and serves as a good indication of the condition of the slave starter motors and the freedom of the engine.

10. Test Data and Calculations

The various correction charts for Dart test, as published in the Rolls-Royce Manuals, are applicable to a wide range of ambient conditions and are accordingly somewhat complex. A study was made of each chart and where practicable the relevant information appertaining to the Mascot locality was extracted and presented in a simple form for use by the test personnel (*see* Figs. 9(*a*) and (*b*)).

A chart was also made giving details of actual r.p.m. and statimeter loads applicable to various standard test conditions at varying A.I.T. (Fig. 10). This ensures that power curves are run over the optimum range and that inadvertent over-powering does not occur. The function of test inspection is taken to include the reduction of all observed figures to standard conditions and the plotting and extraction of data from these figures.

This work is done in the control room and enables testing to proceed without delays from this source, the simplified correction charts, and so on, enabling us to produce the required figures within five minutes of the completion of power curves. The simplification technique was extended to all possible phases of test, a typical example being the over-stamping of the J-60909 over-speed governor setting jig to read directly in r.p.m., instead of degrees. This enables the governor to be set without the necessity of converting r.p.m. to degrees, and results in a considerable saving of man-hours over a run of engines.

Test cards are laid out so that entries are made in the logical sequence of recording, it being possible to take a complete set of results without a break in sequence of either readings or entries.

11. Results to Beginning of January 1958

11.1. TEST STATISTICS

The following data represent all running, including initial calibration work, that has been carried out at Mascot in the first 2,250 hours of T-bench operation, which was achieved by the beginning of January 1958.

Engines passed to service ...	382
Fuel consumed	225,641 gall.
Water-methanol consumed ...	14,113 gall.
Oil consumed	1,300 gall.

DART MAXIMUM POWER CONDITIONS FOR PERFORMANCE CURVES.
CORRECTED r.p.m.

	13,800		14,000		14,500		
A.I.T. °*C*	*Actual r.p.m.*	*Statimeter* (1,165 *b.h.p.*)	*Actual r.p.m.*	*Statimeter* (1,420 *b.h.p.*)	*Actual r.p.m.*	*Statimeter* (1,455 & 1,665 *b.h.p.*)	
5	13,560	807	13,755	1,111	14,245	960	1,258
6	13,585	806	13,780	1,109	14,275	958	1,256
7	13,610	804	13,805	1,107	14,300	956	1,254
—							
—							
23	13,995	783	14,195	1,076	14,700	930	1,219
24	14,020	781	14,220	1,074	14,725	928	1,217
25	14,040	780	14,240	1,073	14,750 (maximum permissible steady r.p.m.)	927	1,215
						Figures below correct to I.S.A. +20°C (1,335 & 1,400 *b.h.p.*)	
26	14,065	778	14,265	1,071	14,290	878	1,054
27	14,085	777	14,290	1,069	14,310	877	1,052
28	14,110	776	14,315	1,068	14,335	875	1,051
—							
—							
43	14,455	757	14,665	1,042	14,690	854	1,025
44	14,475	756	14,685	1,041	14,715	853	1,023
45	14,500	755	14,710	1,039	14,735	852	1,022

Correction for barometer variation from standard ± 2 lb. on statimeter per 0·1 in. Hg.

FIGURE 10. Table of corrected data to ensure ideal power coverage of performance curves.

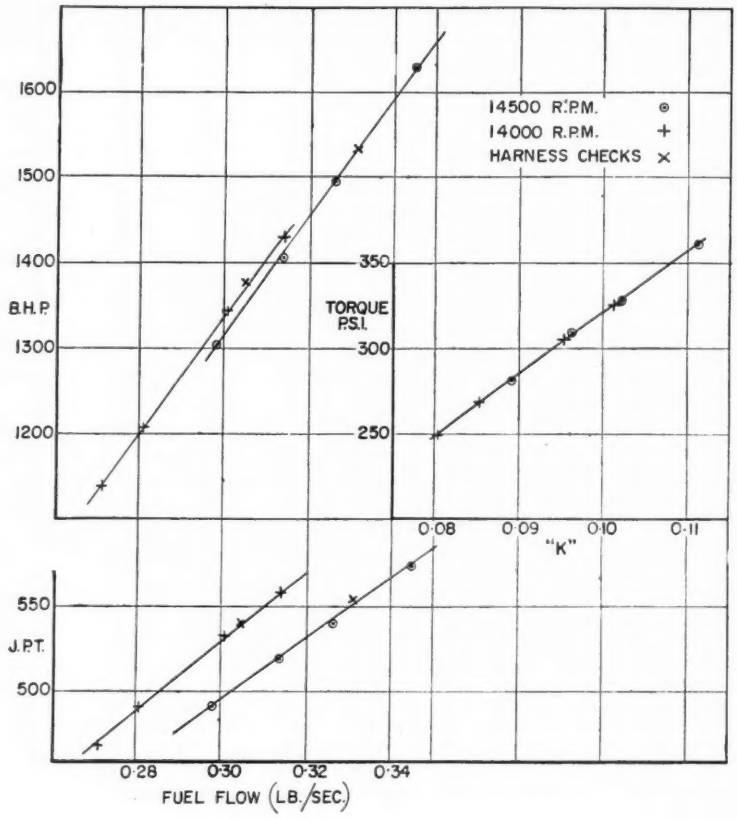


FIGURE 11. Typical T-bench performance curves.

The figures below are averages per engine, extracted from the record of the last 25 complete overhauls.

Fuel	485 gall.
Water-methanol	28 gall.
Oil	23 pts.
Running time	4 hrs. 56 min.

It is interesting to note that, of the grand total of fuel quoted, we have passed approximately 2,000 gallons to the factory oil fired steam generator. This fuel is that recovered from the drain system, which receives all spill, purging, and so on, during both engine mounting and running.

The oil figures include all lubricant lost while removing the propeller, checking filters, and so on. The oil consumed by the engine is less than one pint per test, a figure of 100 cc. per hour being well above average.

Figure 11 shows actual power curves as recorded on the T-bench. It will be seen that scatter due to hysteresis, and so on, is non-existent. Under gusty conditions a certain amount of statimeter "wander" occurs, but it is always easy to determine the point about which this fluctuation takes place, and accordingly record the correct torque. In general, results achieved compare favourably with any other form of dynamometer.

To maintain test records in as concise a form as possible, the reverse side of the power curve sheet lists all relevant test data.

No structural failure of any kind has occurred on the Curran T-bench and it has only been necessary to change the statimeter on two occasions.

The auxiliary loading brake shows no evidence of wear, and trouble in the engine components due to vibration from this source has been non-existent.

The de-mineralisation plant, as well as producing water for the water-methanol mix, is used as the source of the water used in the various process tanks in the overhaul shop. Consumables used in this unit amount to 35 lb. of sodium hydroxide, 6 gall. of commercial hydrochloric acid, and one charge of resins.

11.2. PROPELLERS

Operation began with two 505/506 type propellers. After the first 800 hours had been completed on one of these propellers, it was sent for overhaul and subsequently kept as a spare. Shortly after this time, T.A.A. began operating the Dart 510 which, due to its "third oil line" system needed a different propeller. We accordingly purchased a test fan suitable for this Mark of engine and when, at the beginning of 1957, the majority of T.A.A.'s engines were changing over to the latter type we fitted a set of spare 510 blades into the newly overhauled 506 hub and made the necessary modifications to convert this into the "third oil line" type.

The current position is that we have one 506 propeller recently overhauled and two 510 propellers in operation with 300 and 400 hours running time apiece.

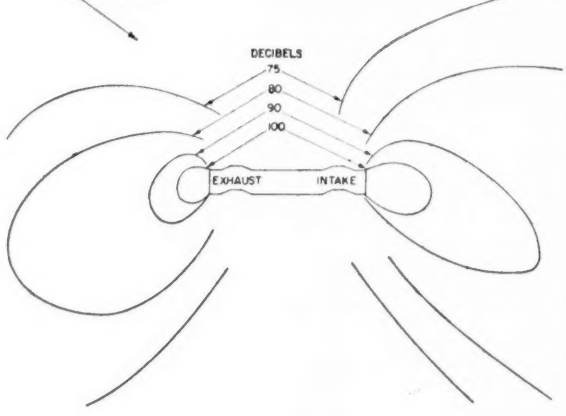


FIGURE 12. Noise levels. Dart 510 at take-off power. Test house 137 ft. × 21 ft. 4 in. overall.

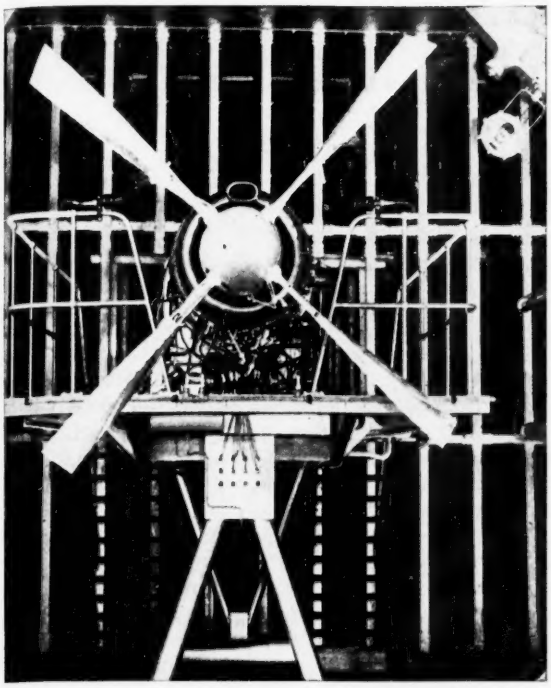


FIGURE 13. Prototype Curran Mk. II T-bench.

11.3. SILENCING

In view of the increasing awareness of the public to the problems of noise associated with gas turbine engines, a survey of the noise level around the Mascot test installation was made with the Dart 510 running at take-off conditions. Fig. 12 shows the results achieved.

It is interesting to note that it is impossible to tell whether an engine is on test or not from the inside of the fitting shop, which is 125 yards distant in a direct line with the cell intake.

11.4. T-BENCH ENGINES IN SERVICE

When the first engines were installed, ground runs showed that "installed coolness" as experienced with dynamometer tested engines was not present and that water-methanol check pressures lined up with hangar settings.

Initially there was a tendency for the throttle clearance checks to be slightly biased within acceptable limits and investigation into this phenomenon led us to modify the throttle control run on the Curran T-bench, as it was found that with the original layout the tendency of the engine to lift when under power was introducing an unwanted variable. It subsequently became normal procedure for the engine to be installed without any control alterations being necessary.

Although there is no direct evidence, it is considered quite feasible that the relatively vibration-free propeller testing, compared with dynamometer lineshaft torsionals, has had a beneficial effect on engine reliability and, in particular, on the low incidence of reduction gear trouble on Australian engines.

12. Future Development

In view of the success of the Australian Curran T-bench, torque reaction equipment of this type was declared in 1955 an officially recommended item of overhaul tooling for Rolls-Royce turbo-prop engines.

Since that date, three T-benches, at L.A.V., Venezuela, C.A.A. Africa and Capital U.S.A., have been commissioned, and were by September 1956 passing their first engines to service.

Other overhaul shops that will be installing or commissioning T-benches in the near future include the following:—New Zealand National Airways Corporation, Air France, Indian Airlines, Hong Kong Aircraft Engineering Company, Philippine Air Lines, Continental (U.S.A.), Aircraft Carriers Engine Services (U.S.A.) and Alfa Romeo (Italy)*.

As a logical follow up of the original 2,500 b.h.p. bench, a Mk. II model designed for engines up to 7,500 shaft horse power has now been produced by John Curran Ltd., and has recently undergone initial calibration tests in the U.K. (*see* Fig. 13).

The Mark II is of somewhat cleaner construction than the original type and incorporates the lessons learned in Australia. With all the development facilities available in the U.K. there is no doubt that this bench will become as great a factor in the overhaul of the coming generation of high powered turbo-prop engines as has the original T-bench for the Rolls-Royce Dart.

*The Hong Kong Aircraft Engineering Co. and Philippine Air Lines T-benches are now in operation.

A Theory of the Cylindrical Ejector Supersonic Propelling Nozzle

by

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Introduction

The so-called ejector nozzle has received a large amount of experimental study as a means for achieving efficient supersonic expansion of the main jet without the difficulties of a mechanically variable convergent-divergent nozzle. No satisfactory theory for the calculation of its performance has yet been put forward and it is the object of this paper partly to remedy this defect.

The ejector nozzle consists of the normal primary nozzle attached to the engine jet pipe, which, in general may be made variable, to provide for reheat; surrounded by a shroud (Fig. 1) whose internal shape is often made somewhat convergent-divergent, though its actual shape is a matter of compromise. The area of the shroud exit in relation to that of the primary nozzle is known as the " area ratio " of the nozzle and is chosen large to match high expansion ratios, and low for moderate ratios. The shroud is fed with secondary air at a lower pressure than the primary, this air being supplied either from the main air intake through a suitable control valve or possibly from a secondary air intake. The pressure of this secondary air virtually controls the amount of expansion of the primary jet. The secondary air thus acts as a form of cushion against which the primary air jet expands. A satisfactory method of calculating the secondary pressure and flow is required in order to predict the thrust of such a nozzle.

Although not necessarily the best, an ejector nozzle with a secondary shroud of parallel cylindrical form has great convenience from the theoretical viewpoint, since in the absence of friction no thrust forces can act on the shroud. This simplifies the theory, and test results show that the performance of such a nozzle is sufficiently good to make a theoretical knowledge of its performance desirable. This paper only deals with such a cylindrical ejector.

Assumptions

Figure 2 shows the cylindrical ejector diagrammatically and the flow quantities indicated.

It is assumed that the primary jet, at total pressure, P_j, issues from the choked primary nozzle and expands as indicated by the diagram, without any mixing with the surrounding secondary air flow. At the secondary nozzle exit it is travelling axially and is at the same static pressure as the secondary air. The primary flow is not isentropic and will in general contain shock waves within its flow.

The secondary air flow is assumed to be isentropic and to follow the contour of the primary jet without loss or mixing. Because of the curved flow there are radial variations in pressure except at exit, but in this paper average values of flow are used and there is no static pressure balance except at the outlet plane 2. Friction losses against the shroud are ignored. Obviously to make these assumptions completely realistic the shroud length must be somewhat delicately chosen.

Notation

P	total pressure
p	static pressure
T	total temperature
t	static temperature
V	gas velocity
m	gas mass flow
A	shroud exit area
A_j	primary jet nozzle area
γ	ratio of specific heats (taken as 1·4)
R	gas constant
M	ratio $\dfrac{V}{\sqrt{(\gamma g R t)}}$ (Mach number)
ρ	density

Suffixes

j	primary flow
s	secondary flow
1	conditions in plane 1, the outlet of the primary nozzle
2	conditions at the outlet of the secondary shroud.

Theory

Because this is a compressible, indeed a supersonic, flow problem, there is little use in expressing all the results in equation form, since no algebraic use can be made of these equations. The method is to use standard air flow curves which are now familiar to most workers in gas dynamics. For convenience, both primary and secondary are assumed to possess the characteristics of air, $\gamma = 1{\cdot}4$.

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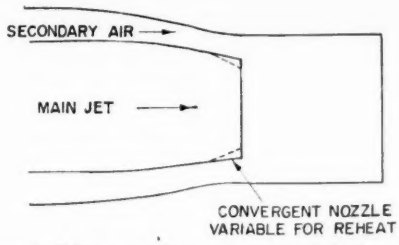

FIGURE 1. Diagrammatic arrangement of ejector nozzle.

The primary flow is assumed to be choked. The primary total pressure and temperature and the primary nozzle exit area are regarded as known quantities, hence the primary flow, m_j, is known; the secondary air total temperature, T_s, and the outlet area, A, are known. It is required, for a fixed ratio of secondary air flow to primary air flow, to calculate the required secondary pressure, P_s, and all the quantities in the outlet plane 2, so that the thrust, and so on, may be calculated.

The problem would be quite calculable directly if the outlet static pressure were assumed atmospheric. In general, this will not be so; it is assumed here that it is greater than this value. When the solution gives a value less than this, then the method is rejected and the atmospheric pressure solution adopted. This makes the method agree with the standard method of calculation of flow in nozzles.

The derivation of the required quantities may be illustrated by the following steps, although actual computation may be carried out in a variety of ways.

To begin with, a value of P_s, the secondary flow total pressure, is assumed. Then a value of the outlet static pressure, $p_{s2} = p_2$, which has the same value for primary and secondary flows, is assumed.

As the secondary air flow is assumed isentropic the ratio, P_s/p_2, enables the flow velocity and density to be calculated in the outlet plane 2. As the total secondary flow is known, the area occupied by this flow is calculated. The flow area occupied by the primary flow is thus obtained. As the primary air flow total temperature, mass flow, static pressure and area are known, all the relevant quantities (velocity, and so on) of the primary flow may be calculated.

This process may be carried out for a range of assumed values of p_2, and the outlet flow quantities are

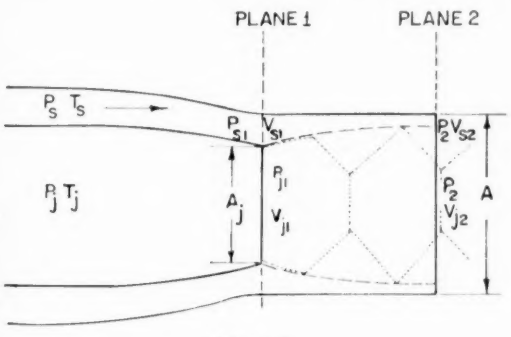

FIGURE 2.

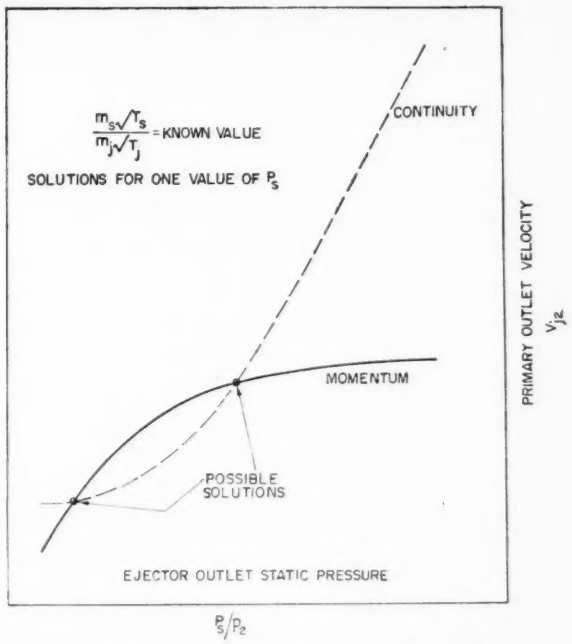

FIGURE 3. Cylindrical ejector nozzle. Possible solutions.

fixed by continuity with the flow quantities in plane 2. Thus, for instance, a plot of V_{j2} against P_s/p_2 may be made as in Fig. 3.

Now there is another relationship which must be obeyed by the flow; momentum must be conserved between planes 1 and 2.

$$i.e. \quad (A - A_j)\,p_{s1} + m_s V_{s1} + A_j p_{j1} + m_j V_{j1} = A p_2 + m_s V_{s2} + m_j V_{j2} \qquad (1)$$

All quantities on the left-hand side are known or directly calculable, while, for an assumed value of p_2, as before V_{s2} may be found. Thus equation (1) gives another value of V_{j2}. For different assumed values of p_2 we get the dotted line of Fig. 3 and where this line intersects the previous curve represents two possible solutions.

In general it will be found that the two curves, representing the two requirements of continuity and momentum, will either give two intersections or will not intersect at all. The latter condition arises from having assumed too low a value for the secondary pressure, P_s, so that the required flow quantity, m_s, cannot be forced through at all. Thus, provided P_s is assumed high enough, there are two possible working points for any P_s and in any case any number of assumed values of P_s. It is the problem of this paper as to how to choose the value of P_s, and this is done by assuming that the ejector will function at the minimum possible value of P_s at which a solution can be obtained. It will readily be seen that this will occur when the two curves of Fig. 3 touch tangentially, and there is then only one possible working point.

This assumption of minimum P_s for a given flow quantity has caused some concern among those with

whom this theory has been discussed. This is particularly so when some of the results of the theory are considered. It is the intention of this paper to justify the assumption or rather its results. Here it may be said that it seems natural that the system should set itself up to require the minimum value of the bypass pressure to pass a given quantity of secondary flow; or stated another way, the system will pass a maximum secondary flow for a given value of the secondary pressure. Again it should be emphasised that this is only so as long as the outlet static pressure is greater than atmospheric. Where this is not so by this solution, the atmospheric pressure at outlet is assumed and the solution becomes perfectly definite without any preliminary assumptions.

Appendix I gives a sample calculation with a typical result. Some results of these calculations are shown later.

Because of the disputed nature of the solution presented here, the implications of the solution are now dealt with.

Type of Solution

In general the static pressure at outlet is found to be in excess of the ambient atmospheric. The velocity of the primary jet in the outlet plane is, of course, supersonic, while that of the secondary flow is subsonic in the outlet plane.

(The term sonic is taken to refer to the flow when $V=\sqrt{(\gamma Rgt)}$ and M is defined as $V/\sqrt{(\gamma Rgt)}$, whatever the actual velocity of wave propagation may be.)

Thus the disputed result is obtained of a jet exit, with a partially subsonic jet emerging at a pressure greater than ambient. It has been asserted that there is nothing to prevent further expansion of this secondary jet to atmospheric pressure. This is not so, however, for if it did, the flow area would contract, but the flow area of the primary would expand by a larger amount, so that the total flow could no longer be contained within the shroud area.

Others argue that the minimum pressure P_s argument is wrong, and that the assumption should be made that the secondary air flow is choked at outlet, the outlet secondary Mach number being unity. As a matter of fact, this was the first assumption and the fact that it appeared to be obviously not in accordance with test results (*see* later) was at once a reason for questioning it.

Undoubtedly where a uniform jet emerges axially from a nozzle at a pressure greater than the ambient it must be assumed that the flow is sonic. Where the jet consists of two non-mixed flows of different total pressure this is not the case however, and it is proposed to show that the conditions will be such that one flow is supersonic (the one with the greater total pressure) while the other is subsonic.

The advocates of the sonic secondary flow base their case on the fact that the velocity in the outlet plane must be equal to the local speed of sound, otherwise there would be nothing to prevent the pressure at outlet falling, and this pressure change being propagated upstream. The authors agree with this argument, but they do not believe the local speed of sound under the conditions is given by the normal formula $\sqrt{(\gamma Rgt)}$, but is modified because the sound propagation is, in effect, taking place in a flow with flexible walls.

In the next section it is proposed to derive the solution by these assumptions and show that it agrees with the previous argument of minimum secondary total pressure.

Wave Velocity Argument

Consider the general case of two streams at a common static pressure, p, and different velocities and temperatures (V_j, T_j and V_s, T_s). Let the velocity of propagation of plane waves in the gases be a_j and a_s respectively, and the areas occupied by the flows be A_j and A_s. As we are dealing here entirely with the outlet plane the subscripts 2 have been omitted for simplicity.

Because the flow is taking place in a tube of constant cross section

$$A_j + A_s = \text{constant}$$

and for the wave velocity in both streams to be fixed in space

$$V_s = a_s \quad \text{and} \quad V_j = a_j.$$

Now imagine each flow to undergo a small change in static pressure as it flows through the sound wave. Then for each flow by continuity:—

$$\frac{\partial A}{A} + \frac{\partial \rho}{\rho} + \frac{\partial V}{V} = 0$$

where ρ is the density

and $$\frac{\partial p}{p} = \gamma \frac{\partial \rho}{\rho}$$ by the gas law,

and by momentum $\partial p = -\rho V \partial V$;

therefore
$$\frac{\partial A}{A} + \frac{1}{\gamma}\frac{\partial p}{p} - \frac{p}{\rho V^2}\frac{\partial p}{p} = 0 \quad . \quad . \quad (3)$$

or
$$\frac{V^2}{Rt} = \frac{\partial p/p}{\dfrac{\partial A}{A} + \dfrac{1}{\gamma}\dfrac{\partial p}{p}} \quad . \quad . \quad . \quad . \quad (4)$$

or
$$\frac{V^2}{\gamma Rt} = \frac{1}{1+\gamma \dfrac{\partial A}{A}\Big/\dfrac{\partial p}{p}} \quad . \quad . \quad . \quad (5)$$

(Note that when $(\partial A/A)/(\partial p/p)$ is zero this equation simplifies to the normal expression for the speed of sound.)

Thus
$$\left.\begin{aligned} M_j^2 &= \frac{1}{1+\gamma \dfrac{\partial A_j}{A_j}\Big/\dfrac{\partial p}{p}} \\ M_s^2 &= \frac{1}{1+\gamma \dfrac{\partial A_s}{A_s}\Big/\dfrac{\partial p}{p}} \end{aligned}\right\} \quad . \quad . \quad (6)$$

the pressures being assumed the same in both streams.

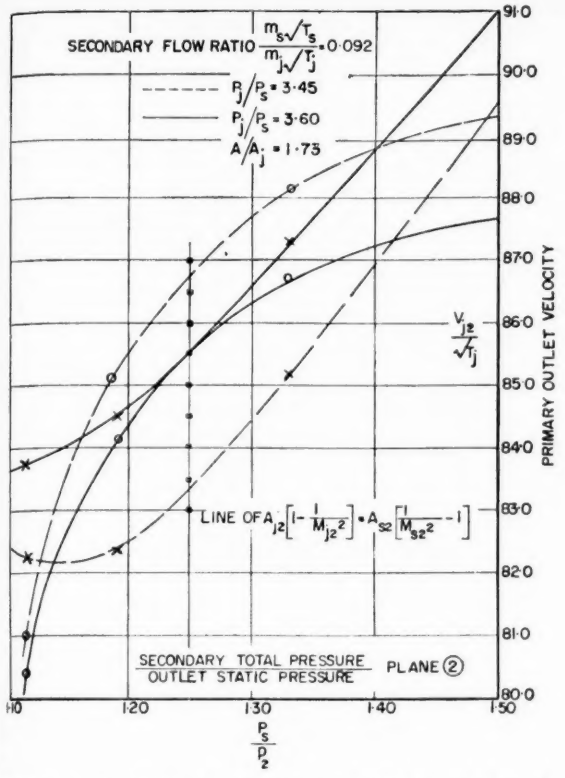

FIGURE 4. Cylindrical ejector nozzle. Theoretical solution of flow equations.

Since, however, $\partial A_j = -\partial A_s$:—

$$\gamma \frac{\partial A_j}{\partial p/p} = A_j\left(\frac{1}{M_j^2}-1\right) = -\gamma \frac{\partial A_s}{\partial p/p} = A_s\left(1-\frac{1}{M_s^2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Hence we obtain the condition, since M_j is regarded as supersonic and M_s subsonic,

$$A_j\left(1-\frac{1}{M_j^2}\right) = A_s\left(\frac{1}{M_s^2}-1\right). \quad . \quad (8)$$

Now there is no obvious relation between this solution and the assumption of minmum P_s. Nevertheless it will be seen from Fig. 4 that the solution given by equation (8) does go through the solution points given by the minimum P_s method.

It will now be shown that minimising the secondary total pressure P_s, or more conveniently optimising the secondary flow for a fixed value of P_s also gives the above Mach number to area relation.

By continuity in the secondary flow:—

$$\frac{\partial A_s}{A_s} + \frac{\partial \rho_s}{\rho_s} + \frac{\partial V_s}{V_s} = 0 \quad . \quad . \quad (9)$$

and as the flow in the secondary is isentropic this becomes:—

$$\frac{\partial A_s}{A_s} + \frac{1}{\gamma}\frac{\partial p}{p} - \frac{1}{\gamma M_s^2}\frac{\partial p}{p} = 0. \quad . \quad (10)$$

For the primary flow which is not isentropic:—

$$\frac{\partial A_j}{A_j} + \frac{\partial \rho_j}{\rho_j} + \frac{\partial V_j}{V_j} = 0 \quad . \quad . \quad (11)$$

and since $\partial A_j + \partial A_s = 0$

$$\text{and} \quad \frac{\partial \rho_j}{\rho_j} = \frac{\partial p}{p} + (\gamma-1)M_j^2\frac{\partial V_j}{V_j}$$

$$\text{therefore} \quad \frac{-\partial A_s}{A_j} + \frac{\partial p}{p} + [(1+(\gamma-1)M_j^2)]\frac{\partial V_j}{V_j} = 0 \quad (12)$$

In addition the momentum in the exit plane 2 must equal that in the primary nozzle plane 1. When the mass flow in the secondary is a maximum the momentum will therefore be a maximum, *i.e.*:—

$$\partial\,[A_j p\,(1+\gamma M_j^2)] + \partial\,[A_s p\,(1+\gamma M_s^2)] = 0 \quad (13)$$

$$i.e. \quad \left[A_j(1+\gamma M_j^2)\frac{\partial p}{p} + 2\gamma A_j M_j \partial M_j + \gamma M_j^2 \partial A_j\right] + \left[A_s(1+\gamma M_s^2)\frac{\partial p}{p} + 2\gamma A_s M_s \partial M_s + \gamma M_s^2 \partial A_s\right] = 0 \quad . \quad . \quad . \quad (14)$$

since $\partial A_j + \partial A_s = 0$.

$$\text{Now} \quad M\partial M = M^2\left(1+\frac{\gamma-1}{2}M^2\right)\frac{\partial V}{V}$$

$$i.e. \quad 2\gamma M_s \partial M_s = -2\left(1+\frac{\gamma-1}{2}M_s^2\right)\frac{\partial p}{p}.$$

With this substitution, and using equation (10), the second bracket of equation (14) reduces to zero.

After some algebra the first bracket of equation (14) simplifies to

$$A_j(1-M_j^2) = A_s M_j^2\left(1-\frac{1}{M_s^2}\right)$$

$$\text{or} \quad A_j\left(1-\frac{1}{M_j^2}\right) = A_s\left(\frac{1}{M_s^2}-1\right).$$

The same relation as previously obtained.

Hence the maximum secondary flow assumption is identical to the conditions for stationary sound waves at the exit of both streams.

Comparison with Experiment

Results of tests on a model nozzle of area ratio $A/A_j = 1{\cdot}73$ and cylindrical secondary shroud are presented in Fig. 5. The nozzle was tested at various fixed values of the (secondary flow/primary flow) ratio and the results are plotted as P_j/P_s against total nozzle pressure ratio P_j/p_a. It will be seen that as the primary pressure ratio increases there is a rapid rise in P_j/P_s, but this ratio rapidly reaches a fixed value after which it does not change. Along the whole of the flat portion of this characteristic the flow pattern in the nozzle is fixed and the system can be regarded as choked.

Theoretical solutions by the methods of this paper give the dotted lines and it will be seen that the agreement is good. The points labelled *a* . . . . . *a* represent the point on the theoretical curves where the outlet plane 2 static pressure is atmospheric. At lower pressure ratios than this, therefore, the solution has

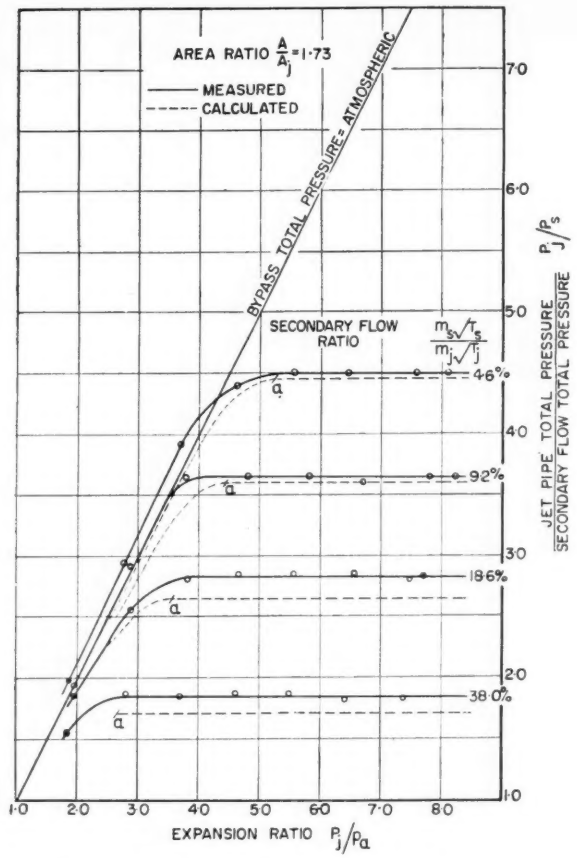

FIGURE 5. Cylindrical ejector nozzle. Secondary flow total pressures.

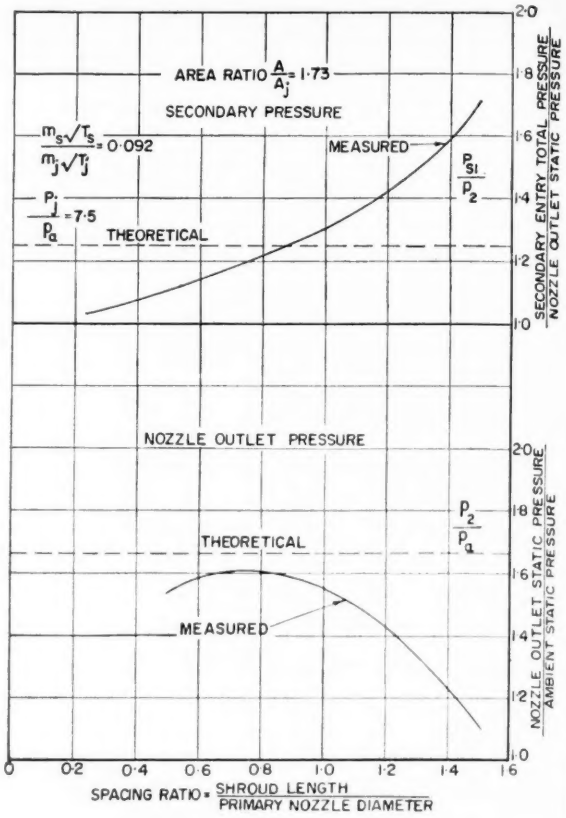

FIGURE 6. Cylindrical ejector nozzle. Comparison of theoretical with measured static pressures.

been worked out on the assumption of atmospheric pressure in plane 2.

These results also serve to show that an assumption of unity Mach number in the outlet plane for the secondary air will certainly not give a good result. For consider the flow ratio of 4·6 per cent. At $P_j/p_a = 8·0$ the value of P_j/P_s is 4·5, therefore, $P_s/p_a = 1·78$ so that there is even at this pressure ratio insufficient secondary pressure to choke the flow. At lower pressure ratios the situation would be even worse, yet the flow is in effect choked down to a pressure ratio of 5·0. The present theory gives satisfactory agreement however.

In our theory mixing has been ignored and the extent to which this simplification affects the result is shown by the curves at low pressure ratios. Clearly in the absence of mixing, P_s must always be greater than p_a, yet the curves do, in part, lie above the line $P_s/p_a = 1·0$. These low values of P_s are due to mixing and to the nozzle tending to work as an injector.

Some results of static pressure measurements along the shroud are shown in Fig. 6. Obviously, if the shroud is too short, the condition of parallel flows at outlet will not be attained, and if the shroud is too long, then further mixing will occur. Mixing will cause the static pressure to fall. The results of these static pressure measurements do, in fact, show that the static pressure is well above atmospheric at the same time that the secondary flow is subsonic. For the longer length shrouds it is evident that mixing is occurring after the point where the primary jet has ceased to expand.

Figure 7 compares thrust measurements made on this nozzle with calculated values. Obviously as agreement was good on pressures the thrust agreement must be fairly close. The curves illustrate the way the nozzle allows supersonic expansion of the primary stream to occur. Curve (A) shows the measured thrust of the whole nozzle, while curve (B) has been derived from it by subtracting the ideal thrust that would be obtained by separately expanding the secondary flow. It will be seen that the primary thrust is in excess of that produced by a normal convergent nozzle, showing that substantial supersonic expansion must have occurred.

The methods of this paper can to a certain extent be applied to the case when the secondary shroud is itself of convergent-divergent form, but a degree of uncertainty is introduced and the best method of application has not been worked out. It is not proposed to deal with this in this paper.

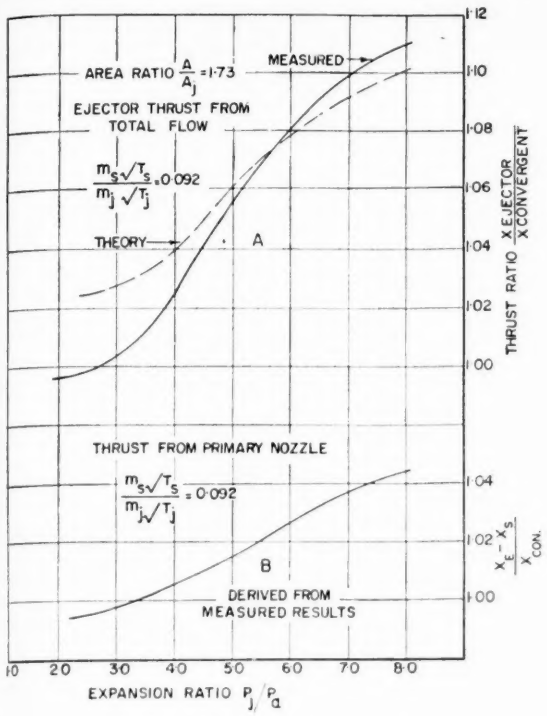

FIGURE 7. Cylindrical ejector nozzle.

Summary and Conclusions

A theory is presented of the ejector nozzle in which the secondary shroud is cylindrical, which is based on continuity, momentum and maximising of the secondary flow. It leads to a result that the overall flow is fixed (or choked) at a condition when the outlet plane static pressure is above atmospheric yet the secondary flow velocity is subsonic as ordinarily defined. The primary flow is, of course, supersonic. This is shown to be a general result where two streams flow through a common nozzle without mixing, whose total pressures are not the same.

The theory is shown to give good agreement with experiment.

APPENDIX I

EXAMPLE IN USE OF THEORY FOR EJECTOR NOZZLE PERFORMANCE

$$\text{AREA RATIO } \frac{A}{A_j} = 1{\cdot}73$$

$$\text{Secondary mass flow ratio } \frac{m_s\sqrt{T_s}}{m_j\sqrt{T_s}} = 0{\cdot}092$$

1. Assume $P_{j1}/P_s = 3{\cdot}6$ $I_{j1} = 3{\cdot}1963$ (critical value from Q curves).
2. Q_{s1} and I_{s1} follow from (1) thus $Q_{s1} = \frac{m_s\sqrt{T_s}}{A_{s1}P_s}$

$$= \frac{m_j\sqrt{T_j}}{A_{j1}P_{j1}} \times \frac{A_{j1}}{A_{s1}} \times \frac{P_{j1}}{P_s} \times \frac{m_s\sqrt{T_s}}{m_j\sqrt{T_j}} = 0{\cdot}3968 \times \frac{A_{j1}}{A_{s1}} \times \frac{P_{j1}}{P_s} \times \frac{m_s\sqrt{T_s}}{m_j\sqrt{T_j}}$$

Proceed by choosing a range of Q_{s2}

(1)

1	2	3	4	5
Calculation of $V_{j2}/\sqrt{T_j}$ *from momentum*				
Q_{s2} →	I_{s2} →	I_{j2} →	Q_{j2}	$V_{j2}/\sqrt{T_j}$
0·3968	3·1963	3·4377	0·301	88·40
0·350	3·4250	3·4166	0·310	86·70
0·300	3·8100	3·3812	0 325	84·14
0·250	4·3900	3·3279	0·347	80·40
0·200	5·3200	3·2423	0·382	70·43

(2)

1	2	3	4	5
Calculation of $V_{j2}/\sqrt{T_j}$ *from continuity*				
P_s/p →	P_{j2}/p	A_{s2}/A_{j1} →	q_{j2} →	$V_{j2}/\sqrt{T_j}$
1·895	6·822	0·3312	1·9350	99·90
1·330	4·788	0·3754	1·4024	87·31
1·192	4·291	0·4380	1·3178	84·50
1·116	4·018	0·5256	1·2901	83·75
1·069	3·847	0·6570	1·4223	87·70

Notes

$$I = \frac{mv}{m\sqrt{Tg}} + \frac{Ap}{m\sqrt{T}} = \frac{1}{g}\frac{V}{\sqrt{T}} + \frac{1}{q} = f(Q)$$

Col. 3 Block 1. $$I_{j2} = I_{j1} + \frac{m_s\sqrt{T_s}}{m_j\sqrt{T_j}}[I_{s1} - I_{s2}]$$

Col. 3 Block 2. $$A_{s2} = \frac{m_j\sqrt{T_j}}{A_{j1}P_{j1}} \times \frac{A_{j1}}{1} \times \frac{P_{j1}}{P_s} \times \frac{1}{Q_{s2}} \times \frac{m_s\sqrt{T_s}}{m_j\sqrt{T_j}}$$

Col. 4 Block 2. $$q_{j2} = \frac{m_j\sqrt{T_j}}{A_{j1}P_{j1}} \times \frac{P_{j1}}{p} \times \frac{A_{j1}}{A_{j2}}$$

$$\frac{A_{j2}}{A_{j1}} = \frac{A}{A_{j1}} - \frac{A_{s2}}{A_{j1}}$$

TECHNICAL NOTES

The Acceleration of a Single Degree of Freedom System Through its Resonant Frequency

by

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IT IS SOMETIMES required to find the maximum amplitudes of vibration attained and the speeds at which they occur when a machine is run through its critical speed with different accelerations. The solution of this problem for single degree of freedom systems has been obtained by Lewis[1] and by Ellington and McCallion[2] for mechanical vibrations and by Hok[3] for the equivalent electrical case. These solutions require higher mathematics (contour integration, Fresnel's integrals or series solutions leading to Bessel functions). The purpose of this note is to show how, by using simple integration only, an alternative method of solution can be obtained for both zero and small values of damping. The method provides the engineer with a physical picture of how the vibrations are produced and shows how solutions may be obtained for accelerations which are nearly, but not exactly, uniform; these are usually close to the solutions obtained for uniform rates of acceleration.

ANALYSIS

It is assumed that the system can be represented by a mass a, on a spring of stiffness c, the damping constant being b. If the displacement is given by x at time t then the equation of motion is

$$a\ddot{x}+b\dot{x}+cx=Pf(t) \quad . \quad . \quad . \quad (1)$$

where $Pf(t)$ is the exciting force the frequency of which varies with time because of the acceleration.

Previous solutions of this problem have been obtained by taking $Pf(t)=P.\sin gt^2$, representing an exciting force the frequency of which increases at a constant rate. In this note, however, it is assumed that the period of the exciting force is decreased by a small amount α after each cycle and that each separate cycle of the exciting force is a true sine wave. The exciting force is thus made up of a series of sine wave cycles the periods of which decrease as indicated above.

The cycles will be numbered from that cycle in which the period of the exciting force is equal to the period of free vibrations of the system, $\frac{2\pi}{\omega_1}$, this cycle being called the "true resonance" cycle. It is thus assumed that a cycle exists in which the periods of the exciting force and the free vibration are equal, but this assumption is for convenience only and does not affect the final result; n will be used to denote the number of cycles above true resonance $\left(i.e.\ \text{cycles with periods} < \frac{2\pi}{\omega_1}\right)$ and m for the number below true resonance.

Hence, for cycles above the true resonance cycle we have

$$\left.\begin{aligned}&\text{period of exciting force} = \frac{2\pi}{\omega_n} = \frac{2\pi}{\omega_1} - n\alpha\\ &\text{and for cycles below the true resonance cycle,}\\ &\text{period of exciting force} = \frac{2\pi}{\omega_m} = \frac{2\pi}{\omega_1} + m\alpha\end{aligned}\right\} \quad . \quad (2)$$

Figure 1 shows the relationship between the period of the exciting force and the number of exciting cycles which have taken place (measured from the true resonance cycle). The curves are drawn for the case of increasing frequency ($Pf(t)\equiv P.\sin gt^2$) and for the case of decreasing period assumed in this note. It can be seen that for the $P.\sin gt^2$ case the curve is a hyperbola and that for the decrease of period case it is a straight line which is arranged to be tangential to the hyperbola at the point where the exciting force period equals the period of free vibrations. In this way the difference between the two assumptions is made to be small over the important range of frequency.

ZERO DAMPING CASE

In this analysis only cycles above true resonance will be considered since the reasoning and mathematics for cycles below true resonance is very similar.

Suppose the system is initially at rest and we apply one sinusoidal exciting cycle of period $\frac{2\pi}{\omega_1}$, equal to the period of free vibrations. Because of this cycle the system will subsequently perform undamped, free vibrations. The amplitude and phase angle relative to a time origin at the start of the exciting cycle can be found by use of Duhamel's Integral. Now immediately after the first

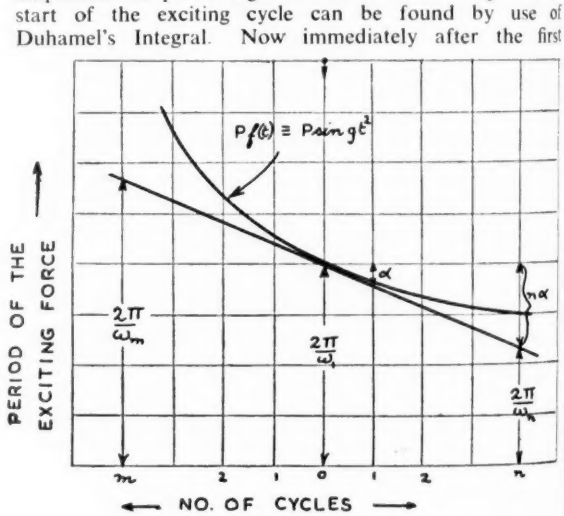

FIGURE 1.

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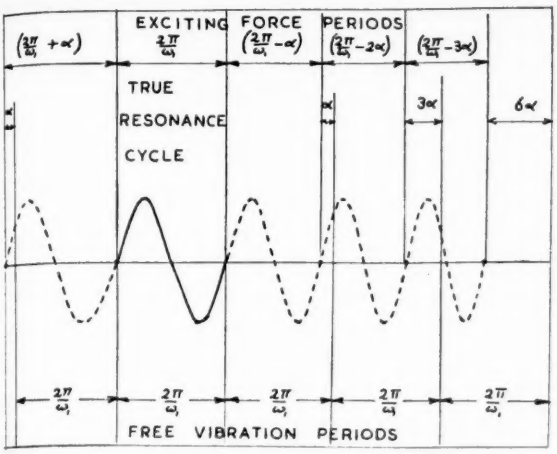

FIGURE 2.

exciting cycle suppose we apply another sinusoidal exciting cycle of period $\left(\frac{2\pi}{\omega_1}-\alpha\right)$. This also produces an undamped, free vibration. We find the amplitude and phase angle of the vibration produced by this cycle, acting alone, as before. This is repeated for n cycles, the period being decreased by α after each cycle. At the end of the n^{th} cycle the total amplitude is the sum of the amplitudes produced by each of the n exciting cycles acting alone and since the phase angles differ the amplitudes must be added vectorially.

Figure 2 shows how the periods of the exciting force cycles decrease compared with the period of free vibrations. It can be seen that the first exciting cycle after true resonance $\left(\text{period } \left\{\frac{2\pi}{\omega_1}-\alpha\right\}\right)$ ends at time α before the end of the first free vibration cycle after true resonance. The second exciting cycle after true resonance $\left(\text{period } \left\{\frac{2\pi}{\omega_1}-2\alpha\right\}\right)$ ends at time 3α before the end of the second free vibration cycle. This time difference is made up of two parts

(*a*) 2α due to the difference in period.

(*b*) α due to the fact that the second exciting cycle starts α before the second free cycle because the first exciting cycle period is α shorter than the first free period.

Similarly the third exciting cycle $\left(\text{period}\left\{\frac{2\pi}{\omega_1}-3\alpha\right\}\right)$ ends 6α before the end of the third free cycle. This 6α is made up of

(*a*) 3α due to the difference in period.

(*b*) $\alpha+2\alpha=3\alpha$ due to the fact that the third exciting cycle starts 3α before the third free cycle because the first and second exciting cycles are shorter than the first and second free cycles by α and 2α respectively.

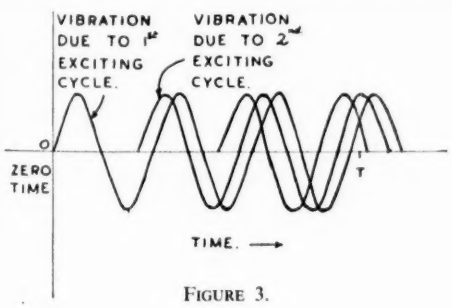

FIGURE 3.

This shows that the total phase angle is made up of two parts, one due to the difference in period and one due to the displacement of the exciting cycle compared with the corresponding free cycle.

It will be shown that the amplitude of the vibration produced by each cycle acting alone is almost constant (independent of the period of the exciting cycle) but that the phase angles of the vibrations are different, *i.e.* they depend on the exciting cycle period.

The method of finding the total resultant amplitude after any exciting cycle is therefore as follows:—

(*a*) Determine the amplitude and phase angle of the undamped, free vibration produced by one sinusoidal exciting cycle of period $\left(\frac{2\pi}{\omega_1}-n\alpha\right)$ for the given α (α is a measure of the acceleration of the system; $2\pi/\alpha\omega_1=q$ is the number of free vibration cycles which take place between the time when the motion starts and the time when the period of the exciting force is equal to the period of the free vibrations).

(*b*) Substituting values of n from one to n' in the expressions obtained above determines the amplitudes and phase angles produced by n' exciting cycles of decreasing period, each acting alone.

(*c*) This is repeated for cycles below true resonance *i.e.* cycles with periods $>\frac{2\pi}{\omega_1}$ *i.e.* $\left(\frac{2\pi}{\omega_1}+m\alpha\right)$.

We have now found the amplitudes (which will all be almost the same) and the phase angles (which will be different) of a number of undamped, free vibrations produced by a number of exciting cycles with progressively decreasing periods. If we now arrange the exciting cycles so that they form a continuous wave of decreasing period the undamped vibrations produced by each cycle are arranged as in Fig. 3. The total amplitude at any time T is the sum of these sine waves.

(*d*) To determine the total amplitude at any time T after, say, N cycles from time zero we must add the amplitudes of the N free vibrations produced by the N exciting cycles on a vector diagram because of the different phase angles. The vector diagram will be of the form shown in Fig. 4. The separate vectors are all drawn to be the same length since the amplitudes of the vibrations produced by all the exciting cycles are almost the same. The point O in Fig. 4 corresponds with time zero on Fig. 3.

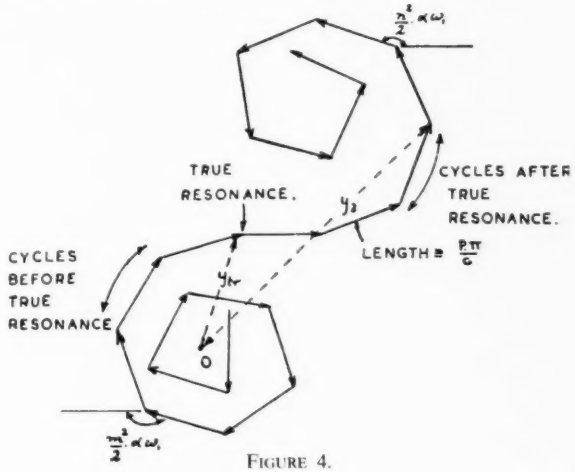

FIGURE 4.

(*e*) To determine the total amplitude after say N exciting cycles from time zero, the length of the line from O to the N^{th} vector is measured on the diagram. For example, the amplitude at the end of the third cycle after true resonance is y_3 on the diagram. In other words when the amplitude is y_3 the ratio

$$\left\{\frac{\text{frequency of exciting cycle}}{\text{frequency of free vibration}}\right\} \text{ is } \frac{2\pi/\omega_1}{2\pi/\omega_1 - 3\alpha} \simeq 1 + \frac{3}{q}$$

From the measurements obtained from the diagram it is possible to plot a graph of the amplitude ratio against the frequency ratio.

The amplitude ratio is

$$\left\{\frac{\text{amplitude at the end of the } N^{th} \text{ exciting cycle}}{\text{static deflection of the system under load } P}\right\}.$$

Hence we must find the amplitude and phase angle of the vibration produced by one sinusoidal exciting cycle of period say $\frac{2\pi}{\omega_n} = \frac{2\pi}{\omega_1} - n\alpha$ acting on a system of natural period $\frac{2\pi}{\omega_1}$.

We do this by using Duhamel's Integral

$$x(t_1) = \int_0^{t_1} f(t)\, h(t_1 - t)\, dt. \quad . \quad . \quad (3)$$

where $x(t_1)$ is the response at time t_1 due to force $f(t)$ at time t. $h(t_1 - t)$ is the response at time t_1 due to unit impulse at time t.

For this case we can say

$$f(t) \equiv P.\sin \omega_n t$$

$$h(t_1 - t) \equiv \frac{1}{a\omega_1} \sin \omega_1 (t_1 - t)$$

$$\text{Therefore } x(t_1) = \int_0^{t_1} \frac{P}{a\omega_1}.\sin \omega_n t \sin[\omega_1 (t_1 - t)]\, dt \quad (4)$$

This has the solution

$$x(t_1) = \frac{P}{2a\omega_1}\left[\frac{\sin \omega_n t_1 + \sin \omega_1 t_1}{\omega_n + \omega_1} - \frac{\sin \omega_n t_1 - \sin \omega_1 t_1}{\omega_n - \omega_1}\right] \quad (5)$$

We require the displacement at the end of the exciting cycle *i.e.* at time $t_1 = \frac{2\pi}{\omega_n}$.

If we put $t_1 = \frac{2\pi}{\omega_n}$ in equation (5) we obtain

$$x = \frac{P}{a}\left(\frac{\omega_n}{\omega_1}\right)\frac{1}{\omega_n^2 - \omega_1^2} \sin \frac{2\pi\omega_1}{\omega_n} \quad . \quad . \quad (6)$$

This is the displacement at the end of the n^{th} forcing cycle after true resonance due to the n^{th} forcing cycle alone.

Similarly the velocity at the end of the exciting cycle is

$$\dot{x}(t_1) = \int_0^{t_1} \frac{P}{a} \sin \omega_n t \cos[\omega_1 (t_1 - t)]\, dt \quad . \quad (7)$$

and at time $t_1 = \frac{2\pi}{\omega_n}$ we have

$$\dot{x} = \frac{P}{a}\frac{\omega_n}{\omega_n^2 - \omega_1^2}\left[\cos \frac{2\pi\omega_1}{\omega_n} - 1\right] \quad . \quad . \quad (8)$$

We now find a sine curve which fits this displacement and velocity and represent it by

$$A \sin(\omega_1 t + \phi)$$

where the time origin is now the time $t = 0$ at which the vibration starts and ϕ is the phase angle between the vibration and a free vibration which starts at time $t = 0$. Hence we have

$$A \sin \phi = x, \qquad A\omega_1 \cos \phi = \dot{x},$$

$$\text{therefore } A = \sqrt{\left\{x^2 + \left(\frac{\dot{x}}{\omega_1}\right)^2\right\}}$$

This gives an amplitude

$$A = \frac{2P}{a}\left(\frac{\omega_n}{\omega_1}\right)\frac{1}{(\omega_n^2 - \omega_1^2)} \sin \frac{\pi\omega_1}{\omega_n} \quad . \quad . \quad (9)$$

and phase angle

$$\phi = \frac{\pi}{2} + \frac{\pi\omega_1}{\omega_n} \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

This is the part of the phase angle which is given by the difference in period.

Now $\frac{2\pi}{\omega_n} = \frac{2\pi}{\omega_1} - n\alpha$, therefore, $\omega_n = \omega_1 \Big/ \left(1 - \frac{n\alpha\omega_1}{2\pi}\right)$

Substituting this in equation (9) we obtain

$$A_n = \frac{2P}{a\omega_1^2}\frac{2\pi}{n\alpha\omega_1}\frac{\left[1 - \frac{n\alpha\omega_1}{2\pi}\right]}{\left[2 - \frac{n\alpha\omega_1}{2\pi}\right]} \sin \frac{n\alpha\omega_1}{2} \quad . \quad (11)$$

A very similar result is obtained for cycles with periods $> \frac{2\pi}{\omega_1}$.

If $n\alpha\omega_1 \ll 2\pi$ these both reduce to

$$A = \frac{P\pi}{a\omega_1^2} = \frac{P\pi}{c}$$

$$= \pi \times \left[\text{static deflection of system under load } P\right].$$

Even if $n\alpha\omega_1$ is as much as $\frac{1}{8} \times 2\pi$ this approximation is still accurate to $3\frac{1}{2}$ per cent. This expression for the amplitude is verified as being sufficiently accurate by the fact that the resonance curves obtained by using this expression agree closely with those given by Lewis (*see* Fig. 5). The method is not accurate for values of $\alpha\omega_1 > 0{\cdot}25$ *i.e.* $q < 25$.

As shown earlier the phase angle is composed of two parts. The part given by the difference in period is

$$\phi_n' = \frac{3\pi}{2} - \frac{n\alpha\omega_1}{2} \text{ for periods } < \frac{2\pi}{\omega_1}$$

$$\phi_m' = \frac{3\pi}{2} + \frac{m\alpha\omega_1}{2} \text{ for periods } > \frac{2\pi}{\omega_1}$$

These have been derived from equation (10) by substituting $\omega_n = \omega_1\left[1 + \frac{n\alpha\omega_1}{2\pi}\right]$ for cycles above true resonance and $\omega_n = \omega_m = \omega_1\left[1 - \frac{m\alpha\omega_1}{2\pi}\right]$ for cycles below true resonance.

To obtain the other part of the phase angle given by the fact that the forcing cycle starts at a different time compared with the corresponding free cycle consider Fig. 2. The time occupied by the first n cycles of the exciting force above true resonance (excluding the true resonance cycle) is

$$\left(\frac{2\pi}{\omega_1} - \alpha\right) + \left(\frac{2\pi}{\omega_1} - 2\alpha\right) + \ldots + \left(\frac{2\pi}{\omega_1} - n\alpha\right)$$

$$= n\frac{2\pi}{\omega_1} - \frac{n(n+1)}{2}\alpha.$$

Hence, since n free cycles occupy time $n\frac{2\pi}{\omega_1}$, the exciting cycle ends $\frac{n(n+1)}{2}\alpha$ before the end of the free cycle. This

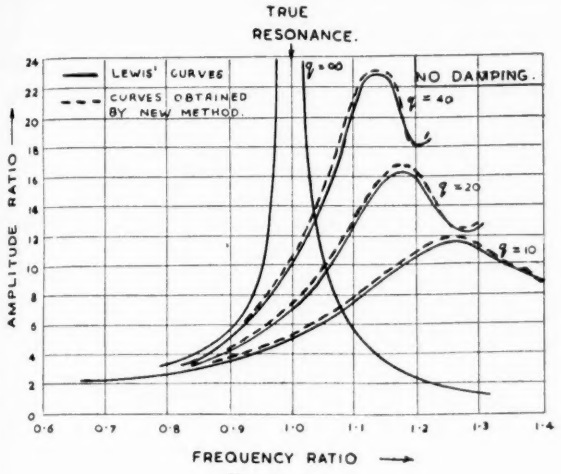

FIGURE 5.

gives an additional phase angle $\frac{n(n+1)}{2}z\omega_1$ for cycles above true resonance. For cycles below true resonance the time occupied by the $(m-1)$ cycles which occur between the end of the m^{th} cycle and the beginning of the true resonance cycle is

$$(m-1)\frac{2\pi}{\omega_1}+\frac{m(m-1)}{2}z$$

and there is therefore an additional phase angle of

$$\frac{m(m-1)}{2}z$$

The total phase angles are therefore

$$\left.\begin{aligned}\frac{3\pi}{2}-\frac{nz\omega_1}{2}+\frac{n(n+1)}{2}z\omega_1&=\frac{3\pi}{2}+\frac{n^2}{2}z\omega_1\\ \frac{3\pi}{2}+\frac{mz\omega_1}{2}+\frac{m(m-1)}{2}z\omega_1&=\frac{3\pi}{2}+\frac{m^2}{2}z\omega_1\end{aligned}\right\}\quad(12)$$

Expressions have now been obtained for the amplitude and phase angle of the free vibration produced by any sinusoidal exciting cycle of given period. Substituting different values of n and m in equations (11) and (12) gives a set of vectors which we combine into a diagram such as that in Fig. 4. Each vector on this diagram corresponds to a vibration produced by an exciting cycle of a given period. For example, the first vector after the true resonance point represents the vibration produced by an exciting cycle of period $(2\pi/\omega_1 - z)$ and the second vector before the true resonance point represents the vibration produced by an exciting cycle of period $(2\pi/\omega_1 + 2z)$. Hence, by taking measurements from zero to any vector, representing the vibration produced by an exciting cycle of period $2\pi/\omega_n$, say, we determine the total amplitude of vibration when the *exciting cycle* has the period $2\pi/\omega_n$.

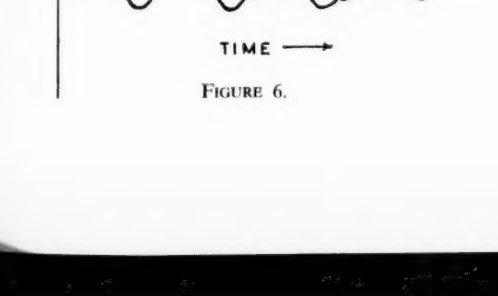

FIGURE 6.

Suppose that the diagram is drawn to such a scale that s units represents $P\pi/c$, the separate constant length vectors. Then if the length on the diagram from zero to the n^{th} vector after true resonance is y_n, the amplitude ratio is

$$\frac{y_n\left(\frac{P\pi}{c}\frac{1}{s}\right)}{P/c}=\frac{\pi y_n}{s}$$

and the frequency ratio is

$$\frac{2\pi/\omega_1}{2\pi/\omega_1-nz}\frown 1+\frac{n}{q}\text{ for cycles above true resonance.}$$

or $$\frac{2\pi/\omega_1}{2\pi/\omega_1+mz}\frown 1-\frac{m}{q}\text{ for cycles below true resonance.}$$

Hence we can plot graphs of amplitude ratio against frequency ratio for different z. This has been done for the same values of z as those used by Lewis. The curves obtained are compared with Lewis's in Fig. 5.

DAMPING

We now introduce damping into the system. Suppose that at time zero we apply one sinusoidal, exciting cycle. This now produces a damped, free vibration which dies away exponentially. If we apply N of these exciting cycles in turn (the period progressively decreasing as in the case of no damping) each cycle produces a damped, free vibration and the total amplitude is the sum of these. We have, in fact, a number of damped, free vibrations arranged in a similar manner to that shown in Fig. 6.

At the end of the N^{th} exciting cycle the total amplitude is composed of a number of amplitudes contributed by the vibrations produced by each exciting cycle. But, whereas in the case of no damping the amplitudes were all the same, in the case of damping the amplitudes due to the later cycles are greater than those due to the earlier cycles. This can be understood when it is realised that, because of the damping the amplitudes of all the vibrations produced by exciting cycles preceding the N^{th} exciting cycle are reduced, the amplitude being reduced more the more distant the exciting cycle (which produced the vibration) is from the N^{th} cycle.

Comparison with Lewis's curves shows that for small values of the damping (*i.e.* $\beta < 0{\cdot}025$ where $\beta = b/b_c$ is the ratio of the actual damping to the critical damping) it is sufficiently accurate to assume that the amplitude at the end of the exciting cycle is the same as in the case of no damping and that after the exciting cycle the amplitude of the vibration produced decreases exponentially as usual for a damped, free vibration. In other words, it is assumed that there is no damping during the time the exciting cycle takes place but immediately after the end of the exciting cycle damping comes into play.

Comparison with Lewis's curves also shows that for larger values of the damping ($0{\cdot}025 < \beta < 0{\cdot}1$) it is necessary to bring in further corrections. It is no longer accurate to assume that there is no damping during the exciting cycle. A term to cover damping during the exciting cycle may be introduced into the Duhamel's Integral and new amplitudes and phase angles found in a very similar manner to that used for zero damping.

Taking account of damping equation (4) becomes

$$x(t_1)=\int_0^{t_1}\frac{P}{a\omega_1}e^{-(b/2a)(t_1-t)}\sin\omega_n t\sin[\omega_1(t_1-t)]\,dt$$

and in a similar way an expression for $\dot{x}(t_1)$ may be obtained.

Solving these for x and $\dot{x}$ at time $t_1 = 2\pi/\omega_n$ and proceeding in the same way as in the case of zero damping we find expressions for the amplitude and phase angles:—

$$\phi_n = \frac{n(n+1)}{2}\pi\omega_1 + \phi_n'$$

$$\phi_m = \frac{m(m-1)}{2}\pi\omega_1 + \phi_m'$$

where $\tan\phi' =$

$$\frac{\left[1+\frac{(\beta\omega_1)^2}{\omega_n^2-\omega_1^2}\right]\sin\frac{2\pi\omega_1}{\omega_n} - \frac{2\beta\omega_1^2}{\omega_n^2-\omega_1^2}\left[e^{b\pi/a\omega_n} - \cos\frac{2\pi\omega_1}{\omega_n}\right]}{\left\{\left[\cos\frac{2\pi\omega_1}{\omega_n} - e^{b\pi/a\omega_n}\right]\left[1-\frac{(\beta\omega_1)^2}{\omega_n^2-\omega_1^2}\right] - \beta\left[1+\frac{2\omega_1^2}{\omega_n^2-\omega_1^2}+\frac{\pi\beta^2}{\omega_n^2-\omega_1^2}\right]\sin\frac{2\pi\omega_1}{\omega_n}\right\}}$$

This reduces to the expression obtained in the case of no damping if β is put equal to zero in this expression.

Substituting $\omega_n = \dfrac{\omega_1}{1-\dfrac{n\pi\omega_1}{2\pi}}$

and $\omega_n = \omega_m = \dfrac{\omega_1}{1+\dfrac{m\pi\omega_1}{2\pi}}$

we obtain $\tan\phi_n' =$

$$\frac{-n\pi\omega_1 + N\beta^2\left[\pi(1+4\pi\beta) - \frac{4\pi^2}{n\pi\omega_1}(1+\pi\beta) - \frac{\pi n\pi\omega_1}{\beta}\right]}{-\frac{(n\pi\omega_1)^2}{2} + N\beta^2\left[\frac{2\pi^2\beta}{n\pi\omega_1}(1+\pi\beta) + \frac{n\pi\omega_1}{\beta}\left\{\frac{2}{N}+\pi\beta\left(\frac{2}{N}+\frac{1}{2}\right)\right\} - \frac{2\pi^2}{N}\right]}$$

and $\tan\phi_m' =$

$$\frac{+m\pi\omega_1 + M\beta^2\left[\pi(1+4\pi\beta) + \frac{4\pi^2}{m\pi\omega_1}(1+\pi\beta) + \frac{\pi m\pi\omega_1}{\beta}\right]}{-\frac{(m\pi\omega_1)^2}{2} - M\beta^2\left[\frac{2\pi^2\beta}{m\pi\omega_1}(1+\pi\beta) + \frac{m\pi\omega_1}{\beta}\left\{\frac{2}{M}+\pi\beta\left(\frac{2}{N}+\frac{1}{2}\right)\right\} + \frac{2\pi^2}{M}\right]}$$

where

$$N = \frac{\left[1-\frac{n\pi\omega_1}{2\pi}\right]^2}{\left[1-\frac{n\pi\omega_1}{4\pi}\right]} \frown 1, \quad M = \frac{\left[1+\frac{m\pi\omega_1}{2\pi}\right]^2}{\left[1+\frac{m\pi\omega_1}{4\pi}\right]} \frown 1.$$

These angles are found to be of the form

$$\phi_n' = \frac{\pi}{2} - \theta, \quad \phi_m' = \frac{\pi}{2} + \psi$$

where θ and ψ are small angles for very small $n\pi\omega_1$

Now $A_n = \sqrt{\left\{x_n^2 + \left(\frac{\dot{x}_n}{\omega_1}\right)^2\right\}} = x_n\sqrt{\left\{1+\left(\frac{\dot{x}_n/\omega_1}{x_n}\right)^2\right\}}$

$$= x_n\sqrt{\left\{1+\left(\frac{1}{\tan\phi_n'}\right)^2\right\}}$$

$$A_m = x_m\sqrt{\left\{1+\left(\frac{1}{\tan\phi_m'}\right)^2\right\}}$$

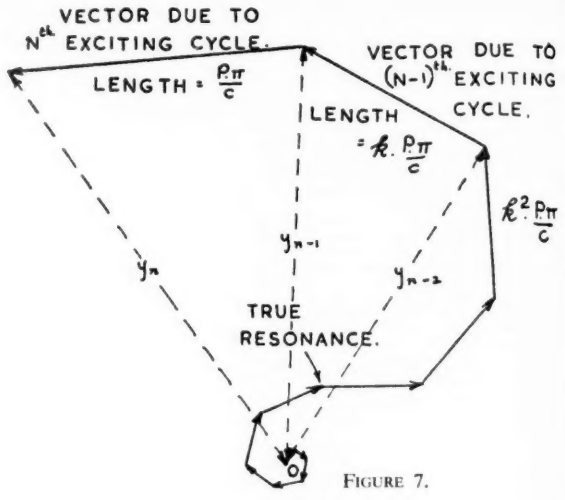

FIGURE 7.

where

$$x_n = \frac{P\pi}{c}\frac{\left[1-\frac{n\pi\omega_1}{2\pi}\right]}{\left[1-\frac{n\pi\omega_1}{4\pi}\right]}\frac{e^{-(2\pi\beta - n\pi\omega_1\beta)}}{\left[\frac{2\pi\beta}{n\pi\omega_1}\left(1-\frac{n\pi\omega_1}{2\pi}\right)\right]^2} \times$$

$$\times\left[1+(1+\pi\beta)N\left(\frac{2\pi\beta}{n\pi\omega_1}\right)^2 + \pi\beta N - \frac{\pi\beta^2}{n\pi\omega_1}(1+4\pi\beta)N\right] \quad (13)$$

and $x_m = \dfrac{P\pi}{c}\dfrac{\left[1+\frac{m\pi\omega_1}{2\pi}\right]}{\left[1+\frac{m\pi\omega_1}{4\pi}\right]}\dfrac{e^{-(2\pi\beta+m\pi\omega_1\beta)}}{\left[\frac{2\pi\beta}{m\pi\omega_1}\left(1+\frac{m\pi\omega_1}{2\pi}\right)\right]^2} \times$

$$\times\left[1+(1+\pi\beta)M\left(\frac{2\pi\beta}{m\pi\omega_1}\right)^2 + \pi\beta M + \frac{\pi\beta^2}{m\pi\omega_1}(1+4\pi\beta)M\right] \quad (14)$$

For small $n\pi\omega_1$ (*i.e.* $< 0{\cdot}3$) these both reduce to

$$A_n = A_m = \frac{P\pi}{c}(1-\pi\beta)$$

These apply for $0{\cdot}025 < \beta < 0{\cdot}1$

Hence using these expressions we can draw vector diagrams as in the case of no damping to determine the total amplitude at any time. The vector diagram will be of the form shown in Fig. 7. This is drawn for conditions at the end of the N^{th} cycle (from time zero).

Consider the case of small damping. In Fig. 7 the amplitude of the vibration produced by the N^{th} exciting cycle at the end of this cycle is $\frown P\pi/c$. At the end of the N^{th} cycle, however, the vibration due to the $(N-1)^{th}$ exciting cycle has completed one damped cycle and so the amplitude of the vibration due to this cycle is $k(P\pi/c)$ at the end of the N^{th} cycle ($k = e^{-2\pi\beta}$). Similarly the vibration due to the $(N-2)^{th}$ cycle has completed two damped, free vibrations by the end of the N^{th} cycle and so its amplitude is $k^2(P\pi/c)$. Hence the total amplitude at the end of the N^{th} exciting cycle is the vectorial sum of N different amplitudes. The total amplitude at the end of the N^{th} cycle is therefore y_n on Fig. 7.

Now consider the end of the $(N-1)^{th}$ exciting cycle. At the end of this cycle the amplitude of the vibration due to this cycle is $P\pi/c$ and the amplitude due to the $(N-2)^{th}$ cycle is $k(P\pi/c)$. On the diagram these are drawn as $k(P\pi/c)$ and $k^2(P\pi/c)$. Hence it can be seen that the diagram as drawn for the end of the N^{th} cycle is too small

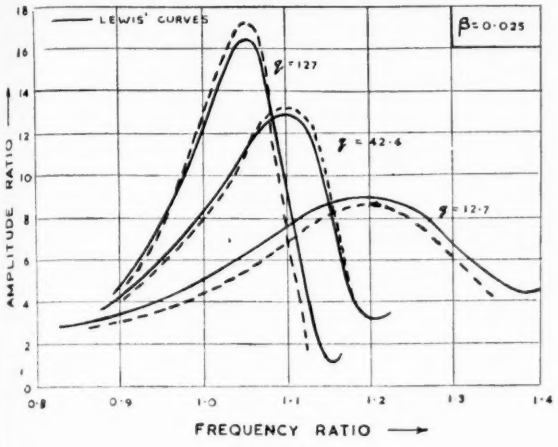

FIGURE 8.

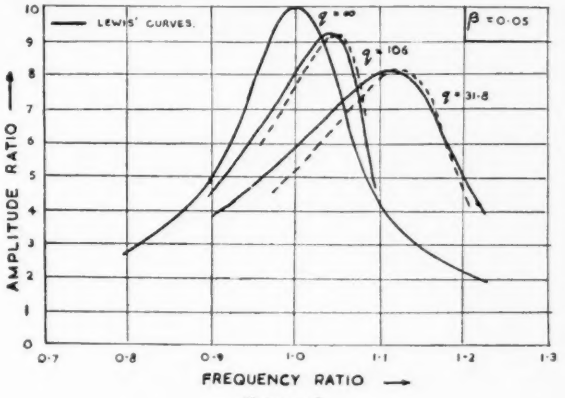

FIGURE 9.

for conditions at the end of the $(N-1)^{th}$ cycle by a factor $1/k$. Hence the total amplitude at the end of the $(N-1)^{th}$ cycle is $\frac{y_{n-1}}{k}$ and not y_{n-1}.

Similarly the amplitude at the end of the $(N-2)^{th}$ cycle is $\frac{y_{n-2}}{k^2}$ and so on.

In the case of larger damping the diagram is very similar to that shown in Fig. 7 but now the phase angles are altered slightly and the amplitude at the end of the exciting cycle is given by equations (13) and (14).

Exactly as in the case of zero damping we can plot graphs of amplitude ratio against frequency ratio. This has been done for the same values of α as Lewis used and both sets of curves are given in Figs. 8 and 9.

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Fail-Safe Structural Design

Comment by

W. T. KOITER

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MR. HARPUR'S interesting and stimulating paper (page 363, May 1958 JOURNAL) is obviously of great importance to all who have to face the problem of fatigue strength of aircraft. These comments all refer to Section 3—Design Loads. Mr. Harpur's conclusions are based on analysis of a typical flight plan of a medium haul civil transport, and are formulated as a proposal for two fail-safe design requirements:—

"(*i*) Where the failure is sufficiently obvious that it will certainly be found in one month at the outside (*e.g.* cracks in the wing bottom skin or fuselage skin) the cracked structure must carry an ultimate load of 44 per cent × 1·5 = 66 per cent of the normal design ultimate, *i.e.* near enough the normal proof taken as an ultimate load.

"(*ii*) When the failure is not obvious and may remain hidden for up to a year (*i.e.* structure inside the wing box, frames and stringers hidden under furnishings in the fuselage) the cracked structure must carry an ultimate load of 55 per cent × 1·5 = 82·5 per cent of the normal design ultimate, *i.e.* near enough 1·25 times the normal proof load taken as an ultimate load.

"If fail-safe tests are carried out some reduction in these requirements is permissible and it is suggested that 80 per cent and 100 per cent of the normal proof loads are taken respectively as ultimate load for the two types of failure (obvious and not so obvious)."

In Mr. Harpur's view these requirements would prove satisfactory for most aircraft and most design cases. Although I am fully aware of the difficulties in the present state of the art to formulate satisfactory design requirements, I am firmly convinced that Mr. Harpur's proposed requirements are inadequate, because they would accept a *far too high probability of catastrophic failure* for the damaged structure, as will be shown in the following.

Mr. Harpur's proposal is based on the assumption that adequate safety of the damaged structure is obtained by applying a factor of safety 1·5 to the loads which occur once (on the average) during the time in which the damage can be detected with certainty. This assumption appears to be based on the satisfactory experience with the factor of safety 1·5 with respect to the limit loading conditions for undamaged aircraft, which loads are supposed to have a frequence of occurrence of the order once in the entire service life of the aircraft.

However, it is easily seen that the proposed requirement (*i*) would classify a structure as "fail-safe" if the probability of catastrophic failure of the damaged structure is of the order of one per cent (the ratio of the time needed in order to detect the fatigue failure with certainty to the

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average time in which a gust in excess of the design gust is encountered, *i.e.* the ratio of 250 hours to 32,000 hours if Fig. 5 of Mr. Harpur's paper is used); this analysis is still unconservative because no allowance is made for uncertainties of load estimation, load distribution and of detail stressing. I am in no doubt that a probability of catastrophic failure of the damaged structure of the order of one per cent is far too high to qualify the structure as "fail-safe."

Moreover, it is suggested by Mr. Harpur that a reduction of the design load by 20 per cent would be permissible if fail-safe tests are carried out. This reduction would even increase the accepted probability of catastrophic failure by a factor of approximately 10, *i.e.* to the order of 10 per cent.

Finally, the analysis, leading to Mr. Harpur's Fig. 5 and the frequency of occurrence of one proof load in 32,000 hours, is rather unconservative. The gust statistics plotted in Fig. 1 appear to yield at 15,000 ft. altitude actual frequencies of gusts between 30 and 60 ft./sec. which are a factor 2 to 3 higher than those plotted in the generalised data of Fig. 2. On the basis of an analysis similar to Fig. 5 it is then found that one proof load occurs on the average every 14,000 to 20,000 hours and the accepted probability of failure of the damaged structure is again approximately doubled.

Hence it must be concluded that Mr. Harpur's proposals would permit an aircraft to be classified as "fail-safe" if the probability of catastrophic failure of the damaged structure is of the order 2 to 20 per cent.

The problem of fatigue strength has been discussed extensively in the past years in the Netherlands Committee on Structural Strength Requirements for Civil Aircraft, established by the Department of Civil Aviation of the Ministry of Transport and Waterstaat. These discussions have recently resulted in a proposal which may perhaps serve as a basis for international discussion on this subject. The chairman of the Netherlands Committee, Professor A. van der Neut, and the Director of the Engineering Division of the Department of Civil Aviation, Mr. C. A. F. Falkenhagen, have kindly permitted me to attach this proposal as an appendix to my comments on Mr. Harpur's paper. It is hoped that this proposal may contribute to a satisfactory solution to the problem of fatigue strength requirements.

APPENDIX

PROPOSED REQUIREMENTS ON FATIGUE STRENGTH

(Prepared by the Netherlands Committee on Structural Strength Requirements for Civil Aircraft, 25th April 1958)

A. *Introduction and explanation of the proposed requirements*

1. In recent years more or less specific requirements on fatigue strength of civil aircraft structures have been introduced in the airworthiness requirements of several states. Due to the complexity of the fatigue problem and the lack of adequate data so far no uniformity has been achieved in these requirements. Nonetheless, agreement seems to have been obtained to some extent on the existence of two basically different approaches to the fatigue problem, characterised respectively by the "safe life" concept and "fail-safe" design, and most effective or proposed requirements give separate treatment to these alternative approaches. However, it is open to doubt whether such entirely separate specification of requirements based on either basic concept is indeed necessary or desirable. It can hardly be denied that predictions on the "safe life" or the "fail-safe" behaviour of a structure are often doubtful. In these cases extensive fatigue testing will be necessary even before a justified choice between the two basic approaches can be made. Moreover, it may happen that a proper combination of the two concepts will lead to a better structure than the application of either single approach. Therefore it would seem worthwhile to attempt the drafting of a more general type of requirement without prejudice to the method in which adequate protection against disastrous fatigue failure is demonstrated. On the other hand, the most general type of requirement, *i.e.* the specification of an allowable probability of a disastrous fatigue failure in service, is too abstruse to be practical in the present state of the art. The present proposals aim at a satisfactory compromise.

2. In drafting the proposed requirements use has been made of available requirements and proposals, more in particular of the U.S. Civil Air Regulations, paragraph 4b.270, of the Australian proposal to the Third Air Navigation Conference of I.C.A.O. (AN Conf/3-WP/39), and of the basic approach by Mr. Lundberg (*e.g.* Proc. 2nd European Aeronautical Congress, Scheveningen (1956), vol. 2, pp. 46.1-46.91).

3. With regard to the U.S. requirements two objections should be raised. First of all the requirements with regard to "fail-safe" do not specify any required fatigue test from which the danger of rapid crack propagation may be assessed. In the absence of such tests it is to be doubted whether a strength requirement for a structure after "fatigue failure or obvious partial failure of a single principal structural element" is adequate.

Moreover, the required static strength after such failure, roughly equivalent to 80 per cent of the static limit loads for the undamaged structure, if necessary increased by 15 per cent in order to account for dynamic effects of failure under static load, seems to be inadequate, even if the failure in question can be detected with certainty before the next flight. In fact, the probability of exceeding these required loads is of the order of 10^{-2} per flight on the basis of available gust data. This probability of failure of the damaged structure is considered far too high to qualify the structure as "fail-safe."

4. The basic idea of the Australian proposal for "fail-safe" requirements of wings is an allowable failure rate of the order of 10^{-8} per flight due to fatigue failure of a single principal structural element *and* the aircraft subsequently striking a gust large enough to break the weakened wing. This seems to be a very sound approach because it carries over a consistent and well-proven level of safety from the static to the fatigue design field. However, the proposed specific requirement, based on this idea, does not seem universally applicable because it is based on a long average life (20,000 hours) of the single element and on the assumption that a failure of this element will be detected with certainty before the next flight. In practice it will be difficult to establish the average life and/or the probability of failure of all "single principal structural elements" in question; besides it will often be impossible to detect such failures with certainty before the next flight.

5. The basic idea of the present proposals, illustrated by Fig. A, is that the probability of a disastrous fatigue failure during the entire service life of the aircraft is of the same order (10^{-4}) as the probability of a failure due to abnormally high static loads.

In order to achieve this object fatigue tests of all critical parts and components shall be made except in so

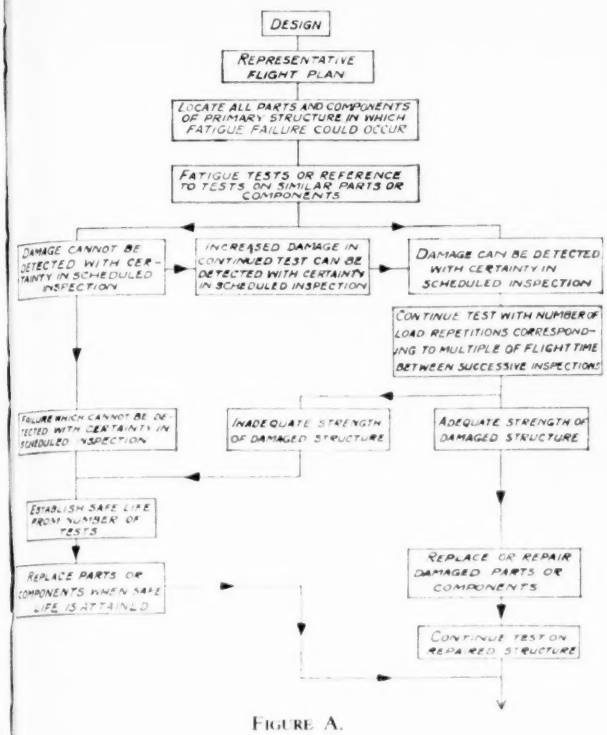
DESIGN
REPRESENTATIVE FLIGHT PLAN
LOCATE ALL PARTS AND COMPONENTS OF PRIMARY STRUCTURE IN WHICH FATIGUE FAILURE COULD OCCUR
FATIGUE TESTS OR REFERENCE TO TESTS ON SIMILAR PARTS OR COMPONENTS
DAMAGE CANNOT BE DETECTED WITH CERTAINTY IN SCHEDULED INSPECTION
INCREASED DAMAGE IN CONTINUED TEST CAN BE DETECTED WITH CERTAINTY IN SCHEDULED INSPECTION
DAMAGE CAN BE DETECTED WITH CERTAINTY IN SCHEDULED INSPECTION
CONTINUE TEST WITH NUMBER OF LOAD REPETITIONS CORRESPONDING TO MULTIPLE OF FLIGHT TIME BETWEEN SUCCESSIVE INSPECTIONS
FAILURE WHICH CANNOT BE DETECTED WITH CERTAINTY IN SCHEDULED INSPECTION
INADEQUATE STRENGTH OF DAMAGED STRUCTURE
ADEQUATE STRENGTH OF DAMAGED STRUCTURE
ESTABLISH SAFE LIFE FROM NUMBER OF TESTS
REPLACE PARTS OR COMPONENTS WHEN SAFE LIFE IS ATTAINED
REPLACE OR REPAIR DAMAGED PARTS OR COMPONENTS
CONTINUE TEST ON REPAIRED STRUCTURE

FIGURE A.

far as reference to tests on similar parts or components may obviate the need of new tests.

The remaining strength of the structure once a fatigue crack or failure has occurred shall be adequate in order that the probability of a disastrous failure before the damage has been detected shall not exceed 10^{-4}.

If this strength requirement cannot be met the part or component in question shall be replaced at such flight time intervals (the safe life) that the probability of an earlier fatigue failure is extremely remote. It might be objected that the safety level defined by these proposed requirements is less than intended (by the probability figure 10^{-4} for a disastrous failure in the entire service life) because fatigue cracks or failures may originate in quite a number of parts or components. On the other hand, once a certain fatigue crack or failure has been observed in one aircraft of a particular type, increased attention will be given to this critical part or component in other aircraft of the same type, and the probability of a disastrous failure due to fatigue of this particular part or component is considerably reduced.

6. In order to account for scatter in the speed of crack propagation the test structure should not be repaired immediately when a fatigue crack occurs which can be detected with certainty in scheduled inspections. It is required in paragraph 7 of the proposed requirements that the test be continued with a suitable multiple of the flight time interval between successive inspections in order to ensure that the damage in actual service will not exceed the damage in the test structure to which the (reduced) static strength requirements are applied. In order to account for possible fatigue failures which may not appear from the tests it is required in paragraph 6 that suitable artificial cracks (*e.g.* saw cuts) shall be introduced in parts where previous experience indicates a likelihood of fatigue cracks in service. It is believed that the present state of the art makes such a necessarily vague requirement inevitable.

7. The required factor of safety for the damaged structure where the damage is such that it can be detected with certainty before the next flight of the aircraft, should be based on a probability of catastrophic failure of the damaged aircraft of the order 10^{-4} per flight. Assuming a flight distance of 1,000 miles, then from available gust statistics the damaged structure should have sufficient strength for gusts up to 65-70 ft./sec. (EAS). Assuming a load factor increase $\Delta n = 1{\cdot}5$ due to a gust velocity of 50 ft./sec. (EAS) at design cruising speed, the required factor of safety is

$$j_1 = \frac{1}{2{\cdot}5}\left[1 + \frac{67{\cdot}5}{50} \times 1{\cdot}5\right] = 1{\cdot}2.$$

This factor of safety may be reduced if the actual cruising speed is considerably below the design cruising speed or if the load factor increase due to a 50 ft./sec. gust is considerably less than 1·5, but it appears unlikely that a factor of safety less than 1·0 would be adequate. The factors of safety j_2 and j_3 for a damaged structure where the damage is such that it can be detected with certainty in scheduled inspections at intervals not exceeding 100 and 2,000 hours may now be obtained by logarithmic interpolation.

$$j_2 = (1{\cdot}5 j_1^2)^{1/3}; \quad j_3 = (1{\cdot}5^2 j_1)^{1/3},$$

resulting for $j_1 = 1{\cdot}2$ in $j_2 = 1{\cdot}3$, $j_3 = 1{\cdot}4$.

8. Whereas it appears from existing requirements and from the present proposals that more or less specific requirements on "fail-safe" structures are feasible, it is unfortunately hardly possible in the present state of the art to draft detailed requirements on the procedure in establishing a "safe life." The required safety factor on life depends on so many variables that a sound engineering judgment is needed in each separate case. It may be emphasised here that the utmost care should be exercised in predicting a "safe life," especially if one-load-level tests are made on separate parts or components. It should be borne in mind that the allowable probability of a disastrous fatigue failure in the "safe life" period shall be extremely remote, *i.e.* of the order 10^{-4}.

9. It should be emphasised that the present proposals need not be more severe than CAR 04b, paragraph 270, although the required factors of safety for the damaged structure are higher. It should be borne in mind that the CAR requirements specify "complete or obvious partial fatigue failure of a single principal element" without any reference to a fatigue test. If the rate of crack propagation is sufficiently slow the present proposals are probably less severe, although adequate in order to ensure the required level of safety. On the other hand, for structures with a rapid crack propagation the CAR requirements are considered inadequate and the (in such cases) more severe proposed requirements are considered inevitable in order to ensure an adequate level of safety.

B. *Proposed requirements on fatigue strength*

1. The strength and fabrication of the aircraft shall be such as to ensure an extremely remote probability of disastrous fatigue failure of the aircraft structure under repeated loads anticipated in operation.

In order to comply with this general requirement the aircraft and/or its components shall be investigated as specified in the following paragraphs. This investigation shall be subject to approval by the competent authorities

2. A *representative flight plan* for the aircraft type shall be defined, and the corresponding operating speeds, weights and external alternating loads shall be determined.

3. The drawings, design calculations and/or strain measurements in a static test shall be scrutinised for the purpose of locating all parts and components in the primary structure in which a fatigue failure could occur.

4. Fatigue tests shall be made on the parts and components referred to in paragraph 3 corresponding to the repeated loads expected in service, except in so far as available test results on similar parts or components under similar repeated loads may be used in lieu of new tests.

5. If a fatigue crack or a fatigue failure occurs in a part or component, tested in accordance with paragraph 4, and if the remaining structure does not comply with the requirements specified in paragraphs 7 to 9, or if such a fatigue failure cannot be detected with certainty in a careful scheduled inspection of the aircraft, a *safe life* of the part or component for the representative flight plan shall be established by means of a suitable factor of safety on life, depending on the accuracy or conservatism of the applied repeated load spectrum, on the number of tests on the same part or component, and on the known or expected scatter in test results.

6. If at any point of time in a test in accordance with paragraph 4 a fatigue crack or failure occurs, and if such a crack or failure can be detected with certainty in a planned scheduled inspection of the aircraft, the test may be continued, subject to the requirements of paragraphs 7 to 10, in order to prove a safe behaviour of the damaged aircraft structure. If during a test several cracks or failures occur independently the following requirements shall be applied to each separate crack or failure. In order to allow for the possibility that not all cracks occurring in service will occur in the test, suitable artificial cracks (*e.g.* saw cuts) shall be introduced in parts where a likelihood of fatigue cracks in service is apparent on the basis of previous experience.

7. The test with the damaged part or component is continued after the point of time mentioned in paragraph 6 with a number of load repetitions corresponding to a suitable multiple of the flight time interval between two scheduled inspections of the type referred to in paragraph 6.

If at any point of time during the continued test a more serious crack or failure occurs which can be detected with certainty in a simpler and more frequent scheduled inspection of the aircraft, the test need only be continued after the latter point of time with a number of load repetitions corresponding to a suitable multiple of the flight time interval between two scheduled inspections of the latter, simpler and more frequent type.

8. The aircraft structure with a damaged part or component, corresponding to the damage at the end of the test in accordance with paragraph 7, shall not show excessive deformations or suffer an excessive reduction in stiffness which might imperil its flight characteristics, control or safe operation, and shall have adequate strength in order to ensure an extremely remote probability of a disastrous failure under static loads.

9. In order to show compliance with the strength requirement of paragraph 8 the remaining primary structure shall have an ultimate factor of safety with respect to the limit loads in the flight conditions and landing conditions depending on the flight time interval between two scheduled inspections of the type in which the damage can be detected with certainty.

10. After completion of the test described in paragraph 7 the damaged part or component may be replaced or repaired in order to prove a safe behaviour of the repaired structure. The test is continued, if necessary, with scheduled replacement of parts referred to in paragraph 5 when they have attained their safe life, until a safe behaviour of the structure has been demonstrated for the entire expected service life of the aircraft.

REPLY BY MR. HARPUR

Professor Koiter's contribution is most interesting, in particular the proposals for fatigue strength requirements put forward in the Appendix. I should point out that I merely intended to indicate in my paper how fail-safe design loads might be fixed and at this stage I should perhaps withdraw and leave it to the airworthiness authorities to settle on a standard of safety. However, having made some specific suggestions in the paper I should explain what I had in mind at the time.

The probability of 1/100 which Professor Koiter questions is, of course, only the probability of hitting a gust during a given period. The probability of failure is then this figure multiplied by the probability of there being a crack of critical length actually present during this period and this is a much lower figure. If the crack is due to fatigue this latter probability increases with flying hours or flights. If the crack is due to corrosion, it also increases, but possibly with calender time only, while if the crack is due to accidental damage, the probability is substantially constant. The length of the crack is a function of the time and length at initiation and of the rate of propagation, if any, from this time.

In fact the three parameters, load, length and time are all interdependent and it is strictly incorrect to define fail-safe requirements in terms of each separately. At the moment the official view seems to be to define the load as a mandatory figure, the length as a recommended figure and the inspection period not at all. The proposals put forward by the Netherlands Department of Civil Aviation recognise this and refrain from defining each parameter separately. In particular, they do not, I am pleased to see, talk about "single principal structural elements" which are very difficult to define. After all, an aircraft with a single spar wing can be just as fail-safe as one with three spars if the rate of propagation of cracks is low enough and inspection reasonably easy.

On the subject of the gust data shown in Fig. 1, I do agree that at the higher gust intensities the data has a wide scatter and that therefore the curve of Fig. 7 is also subject to scatter. The errors will be larger for the larger loads so that while one month is probably still about right for 44 per cent of ultimate, the time for 66 per cent could vary perhaps between 5 and 20 years. What is required is more reliable frequency data for the higher gust intensities.

Comment by

A. H. CRAWSHAW, A.F.R.Ae.S.
(Blackburn and General Aircraft Ltd.)

IN his timely paper on Fail-Safe Structural Design (MAY JOURNAL) Mr. Harpur concludes that 66 per cent of the ultimate strength should remain after an obvious failure of a critical element and 82½ per cent when the failure is hidden. Shall we make it 75 per cent and try to avoid anything being completely hidden? These are criteria which should be debated and agreed.

I should like to ask him, however, whether he is satisfied with the standard of strength to start with?

Referring to Fig. 7 the ultimate design load (4·2*g*) will occur once in 10^7 hours. This means that an airline with 200 aircraft can expect one of these to break up and kill all on board. Is this good enough? Note that this is according to the law of probability. The law of cussedness can cause this to happen to one of the first few.

Again on page 365 (MAY JOURNAL)—"Without any attempt to make it do so, it seems that this particular example fits in with the usually accepted idea that proof load is the load that may be expected to occur once in the life of the aircraft. Then a factor of 1·5 is put on this load to cover uncertainties of load estimation and distribution and of detail stressing." Do we in the Aircraft Industry really think that a factor of 1·5 is sufficient on an actual load that is going to occur on the average once to each aircraft? I do not think any mechanical engineer would think that it is.

In the quotation given the factor of 1·5 is to allow for "uncertainties of load estimation and detail stressing." What about the other items mentioned on page 363, namely, "Damage can arise due to corrosion, stones flying off the runway, punctures from tools or trestles in maintenance or ground handling, failures of radio aerials, turbine discs or propeller blades and even mid-air collisions with birds or other aircraft." I would add bad material and workmanship.

It is stated on page 363 (*iii*) "An adequate reserve of strength in the damaged structure so that, during the period between inspections in which the damage lies undetected, ultimate failure of the structure as a whole is remote." The policy of the industry has been to eliminate reserves of strength as far as possible. The strength required has been cut to what was thought to be an acceptable minimum to reduce weight.

We are still asked to have "adequate reserves" after failure! So an increase in the original strength is required. The quotation from Leonardo da Vinci is interesting; it seems he intended to have 100 per cent of the required strength left after a failure of a critical element.

I, like Dr. Walker, do not want to fly in an aeroplane with a cracked wing. Is there anyone who does?

What has happened is this. The industry has manufactured and sold a large number of aircraft with wings of inadequate strength. Since experience is very long term we are now being faced with many failures. Luckily most of these have been minor. A system of organised repair has grown up and now we are trying to make a virtue out of it. Some are even arguing that if we have adequate "fail-safe" we can reduce strength still further! This would, of course, multiply the number of minor failures.

Our aim should be *no* failures and *also* fail-safe. I feel that ultimate factors of 4 or 4½ will not be sufficient.

Received 1st June 1958

REPLY BY MR. HARPUR

I do agree with Mr. Crawshaw that there is much to be debated and agreed before we can say we have put fail-safe design requirements on the same basis as most of the others we work to. However, until recently they have not been on any basis at all so I think we are improving.

Taking the points he raised in order, I would certainly agree that we should avoid anything being completely hidden from any means of inspection, visual, X-ray or the rest. The figure of 82½ per cent referred to components which would not be seen very frequently but I hope there will be nothing which will never be seen.

The ultimate design load in Fig. 7 is, of course, based on data regarding the frequency of occurrence of the highest magnitude gusts and this is, in its nature, very meagre. It is quite possible, therefore, that the figure of just over 10^7 hours for ultimate load may be too severe. I think this is probably the case because the accident rate due to structural failure under ultimate load is very small, certainly not as bad as would be consistent with a figure of one in 10^7 hours. Fig. 7 is much more accurate, of course, at the low end of the load range.

I think 1·5 is quite an adequate factor on the load which is going to occur once in the life of each aircraft. What is important is the safety factor we have on the load which will occur once in the lifetime of a whole fleet or generation of aircraft and this obviously should not get too near unity. In this respect, of course, it is probably true to say that as the number of aircraft flying around the world increases, the strength requirements should also be increased, so that the number of newspaper headlines relating to crashes remains constant.

On the subject of what is an adequate reserve of strength, this is bound up with inspection periods. The object is to keep the same level of safety always and this can be achieved at a reduced level of strength for a damaged structure if the structure is only going to remain in this state for a limited period of time. This is an accepted airworthiness principle and many examples of it may be found in the requirements. For instance, the design gust intensity of 50 ft./sec. at cruising speed is reduced to 25 ft./sec. at diving speed on the basis that the time actually spent at diving speed is much less, and the chances of hitting the highest gust are substantially reduced. Likewise the vertical velocity of descent for emergency landings at take-off weight is reduced below the value used at the normal landing weight. There are numerous other instances of this quite logical policy of reducing individual levels when considering unlikely combinations of events.

It should be emphasised in answer to Mr. Crawshaw's final point that it is not intended that an aircraft should be allowed to continue flying with a full load of passengers *after* the discovery of a defect. What is being done is to make sure that the aircraft is quite safe *before* the defect has been discovered. While agreeing that no-one likes to fly in a cracked aircraft, it is extremely comforting to know that the aircraft will still hold together even if there are cracks. For cracks in aircraft are inevitable, all types suffer from them and all we can hope to do is to reduce them to the minimum. They can never be completely eliminated. After all, people are quite prepared to sail in a ship even though most ships in the world leak and sink if the water is not continuously pumped out of the bilges. The accent is on control rather than prevention.

Graduates' and Students' Section

Visit to de Havilland Aircraft, Hatfield

While passing through the works we were impressed by the way the space was laid out, with plenty of room, surprising cleanliness, and ample daylight.

de Havilland have long been users of the Redux bonding system, but they have developed their own method of operation, which is kept a guarded trade secret. However, this method of construction was noted in many parts around the assembly line and always allowed a good clean surface finish.

We were shown the glass fibre and Perspex department which is reputed to be the largest in any aircraft company in this country. There they produce a variety of components, from such large items as Sea Vixen and Javelin nose radomes down to small cleats, and from fighter canopys to lamp covers. The resin used to impregnate the glass fibre was of the " hot set " variety; lamps being used as the heat source.

Other places visited included the foundry and press shops, automatic machine shop, and the treatment department. In the machine shop we were shown how rivets are produced, and how nuts and bolts are turned.

Courtesy de Havilland Aircraft Co. Ltd.

Inside the Flight Test Hangar at Hatfield: a Comet is made ready for a test flight.

On the assembly line great care is taken to avoid any scratching of the finished surfaces, and a high standard of workmanship and general interest in the product was refreshing to see.

From the assembly line we moved to the Flight Test hangar where four Comets, an Otter, a Dove, a SAAB Safir and a D.H. Cirrus Moth were housed. Also in this hangar was the Turbi two-seater built by a group of D.H. enthusiasts. It had recently made its first flight in the hands of Pat Fillingham.

It was interesting to compare the various Comets as they were of vastly different types, there being a Comet I looking very small beside two Comet 4's in B.O.A.C. colours, one just back from noise trials (or quietness trials as D.H. prefer to put it) in New York, and a Comet 3B in B.E.A. colours, which is the 3 prototype with its wing cropped by 7 ft. to 4B standard, and liberally tufted with nylon on the upper wing surfaces. All except the Comet I were fitted with late model Rolls-Royce Avons with R.R. pattern noise suppressors in the jet pipes.

Finally we were shown the museum where there are models of all D.H. aircraft and many photographs of significant occasions and general interest.

Future visits

On Saturday 22nd November there will be a visit to the National Physical Laboratory at Teddington. There are many Engineering and Scientific research facilities there that should prove interesting, including a large aeronautical section.

A further visit, to Handley Page Ltd., is fixed for 20th December. Both the Cricklewood and Radlett factories will be shown, and we will be able to see both the Herald and Victor. A coach will transport us between the two places and the visit will start and finish at Cricklewood.

All interested should get in touch with the Hon. Visits Sec., Mr. A. R. M. Pickering at " Riverside," Vicarage Walk, Bray, Berks.

Representative

Mr. V. Morples has kindly agreed to act as our representative in the Oxford area. His address is c/o Engineering Laboratories, 19 Parkes Road, Oxford.

A cheap evening out—Winter Dance

Friday 28th November, is the date of the Section's winter dance; it will be held as usual in the library at No. 4 Hamilton Place between 8 p.m. and 11.45 p.m. Tickets cost a bargain price of 7s. This includes free wine, beer and refreshments! Dancing will be to Arthur Young and his band.

Tickets can be obtained from all Committee members and local representatives or by post from J. R. Cownie, 40 Sherwood Avenue, Marshalswick, St. Albans, Herts.

THE LIBRARY

Reviews

DANGER IN THE AIR. *Oliver Stewart. Routledge and Kegan Paul. London, 1958. 194 pp. Illustrated. 28s.*

This book is an unusual study of a most important subject. Apparently largely based on official accident reports—sometimes amplified by less reliable press information and personal observations and opinions—it sets out to examine in considerable detail a wide variety of air accidents ranging from the loss of the R.101 airship to the Comet disasters. Actually, the R.101 accident seems a bit out of place. It is very much earlier than any of the other accidents considered in detail and, in any case, is of purely historical interest now that the rigid airship has become extinct. All the other accidents, on the other hand, concern heavier-than-air craft and are more recent. They therefore relate much more closely to the problems and circumstances of today's air operations.

Anybody who, like the reviewer, has been involved in matters of air safety and accident investigation will disagree with some of Major Stewart's contentions and conclusions about particular accidents, but it would not be appropriate, nor is there sufficient space in a short notice such as this, for detailed argument about points of disagreement. All that can be said is that Oliver Stewart is as irritating and as stimulating as usual in his forceful and lucid propounding of arguments about causes and effects, many of which he has been advancing for a long time.

It is a pity that this book could not have been written by somebody who is less partisan in his views about various controversial aviation issues and who is prepared to approach with complete objectivity a subject which cannot be usefully discussed on a basis of prejudice or preconceived ideas. It would also have been easier to read if the author had used units with which the majority of his readers is likely to be most familiar! However, few will doubt that, if the book *had* been written by another, it would not have been so well written nor would we have had the benefit of hearing once again in such amusing terms that pilots are always unfairly blamed for accidents, that airline passengers should have parachutes, that flying boats are better than landplanes and that it is only because the airlines are inefficient and have too many ground-borne officials that they cram their passengers into dangerously tight accommodation. No doubt there is some truth in some of these contentions, but there are also unquestionably more important truths—so far as air safety is concerned—which could have been brought out about many other aspects of the difficult subject discussed.

Danger In The Air might have been less readable but it would certainly have been more educational if its author had tried—just a little!—to be as objective as those admirable and dedicated people who spend their lives investigating aircraft accidents.—PETER W. BROOKS.

VIBRATION AND IMPACT. *R. Burton. Addison-Wesley Publications, Massachusetts, 1958. 310 pp. Illustrated. 63s.*

In his book, Professor Burton remarks that "the reader may be dismayed to find that as late as Chapter 12 he will be trying to master derivations that were carried out before 1800. It is a peculiarity of this field that, at an early date, leading thinkers did work which is still of significance today." He is quite right. Much of the theoretical basis of mechanical vibration was well understood years ago. Yet engineering projects are still bedevilled by vibration problems.

It is not difficult to find several reasons why this state of affairs should exist. It is, for instance, one thing to make deductions as to the properties of the familiar inertia and stability coefficients (a_{rs} and c_{rs}) and quite another to affix their numerical values in a given system. Again, it does not follow that, because a body of theory is tolerably complete, it is necessarily convenient to use.

This reviewer believes that another weakness of vibration theory lies in the manner in which it is presented in engineering textbooks—it is not that it is bad, but it does seem stereotyped. As with *Strength of Materials*, a fashion has been set by one or two exceptionally good writers; and the method of attack that these pioneers have used has been largely adopted by others. There seems thus to have grown up a sort of *orthodoxy* where none should exist. One upshot of this is that an undiscerning student may be led by some of the more recent authors to an obsession with rigid masses on light springs. It leads to statements like that on page 210 to the effect that ". . . the effective first-mode spring constant acting on (!) each particle must be smaller than the effective spring constant associated with any other mode. . . ."

Much of this book follows a well-trodden path and is very elementary. In one or two places, the author introduces points of novelty, though always in an elementary form. Thus there is a simple chapter on impact (with very little emphasis on what happens during real impacts), an effort is made to present the elements of non-linear vibration theory in one chapter; again an introduction to matrix methods is included, though the reviewer did not find the lucidity here that one looks for in an introductory textbook. There are also chapters on Waves, Control Systems and Fatigue.

As an elementary introduction for undergraduates, there is much to be said for this book—particularly its breadth of coverage. But it will probably appeal most for recommendation to the weaker brethren. It seems sometimes to lack the profundity and precision that will stimulate and attract the more scholarly. One example of this has been quoted. Another example, which appears on page 211 will perhaps provide a clearer illustration. The author states that "From these considerations (of a matrix iteration) it is seen that the rate of convergence towards the first mode depends upon the relative frequencies of the first and higher modes. If the first and second modes have identical frequencies, they will have identical effective spring constants and the iteration will never converge to either." Now if the first and second modes of a system have equal frequencies, then those two modes are not uniquely defined for the system. The problems that attend this set of circumstances not only appear to be of very great practical importance, but also to raise considerable difficulties. Lord Rayleigh recognised the physical importance of small departures from this "equal-frequency" condition more than half a century ago. With some students, there is perhaps a case for glossing over a matter like this—but not with the better men.—R. E. D. BISHOP.

HELICOPTERS AND AUTOGYROS OF THE WORLD. *Paul Lambermont and Anthony Pirie. Cassell, London, 1958. 255 pp. Illustrated. 30s.*

This book sets out to give descriptive notes and tabulated data of every helicopter built in every country in

the world since Cornu's first successful machine in 1907. Not content with this formidable task the authors have also included quite a number of helicopters which though they did not get beyond the project stage were of interest and importance in the general line of development of a series.

The authors have been remarkably successful in their efforts and the result is a book which will be of the greatest interest and value to engineers and others of both the fixed and rotary wing camps. Perhaps the most useful parts of the book are the general notes about each aircraft, which have a great deal of useful background and historical information about the machine as well as technical description. Unfortunately, because of the range of the book the tabulated data on each aircraft is necessarily somewhat scanty and is confined to principal dimensions, weights and performance. This lack of detailed information makes the book less valuable and the searcher after complete information will have to use a further reference.

However, to obtain and include in the book this additional information for all the aircraft mentioned would involve the authors in a considerably more difficult task and the wonder is that so much information has been gathered about many aircraft which have long been defunct. It is a pity that the range of aircraft covered has on occasions led to some inaccuracies, mainly slight and usually concerned with the background or designation of a particular machine. It is to be hoped that these inaccuracies will be removed in the next edition. A further improvement would be a clear statement of the flight and production status of each helicopter. It is often impossible to discover from the text whether the aircraft has actually been built or flown and in what quantities, and unless he already knows the answers, the reader is left literally, as opposed to actually, in the air.

The 133 photographs are mainly of good quality though not outstandingly original. However, if illustration space was short, it is a pity the authors were not more selective. For example, no fewer than seven photographs of Bell 47 variants appear compared with one photograph of a museum bound Avro-built Cierva Autogiro.

The price of 30s. is modest for a book of this standard and currently offers some of the best value for money in the aeronautical reference book field.—P. A. HEARNE.

DIGITAL COMPUTER PROGRAMMING. *D. D. McCracken. Chapman and Hall, London, 1958. 253 pp. Illustrated. 62s.*

The purpose of this book is to give an entirely self-contained account of the technique of programming, understandable without any previous knowledge of computing. To do this the author has devised (on paper) a computer which has many typical features. In the description of these features other alternative possibilities are discussed and the book thus contains much information on the details of actual computers. The paper computer has a single address order code and is used throughout to illustrate basic techniques for such codes. Although the author has taken care to explain at length points most beginners find difficult, such as the handling of the position of the decimal point, he has not confined himself to elementary topics and includes chapters on interpretive schemes and automatic coding. A particularly helpful chapter is that on getting programmes right.

The appendices include an account of optimum coding. It should have been mentioned here that the full advantage of this is not to be obtained with a one-address code. A serial optimally-coded machine would preferably have a three-address code.

For the person learning to programme nothing can replace the experience of getting programmes to work on an actual machine, but since this book does not in any way minimise the difficulties to be faced it will provide a very useful background for anyone about to study the details of a specific computer. To encourage him most of the chapters are followed by realistic exercises and there is a bibliography.—G. G. ALWAY.

AERODYNAMISCHE PROFILE. *F. W. Riegels. Oldenbourg, München, Germany, 1958. (In German.) 278 pp. Illustrated. 138 DM.*

The author, now head of the department for theoretical aerodynamics at the Max Planck Institut für Strömungsforschung at Göttingen, is known for his theoretical work on aerofoil sections. In this country, attention is drawn to his solution for calculating pressure distributions over thick aerofoil sections at incidence, in potential, incompressible flow, discussed and extended to sheared infinite wings, in R.A.E. Report Aero 2497.

The book, the first comprehensive publication in Germany on aerofoil sections since the *Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen* (1921-1932), presents experimental results for the past 30 years, including German results hitherto unpublished and relevant non-German data. The information is well selected and essential data are tabulated, with derivations; other data are presented in the form of diagrams. Each of the ten chapters of text gives a list of literature on the subject considered, without classified reference.

Although the book, as appears from its title, may be taken as a collection of aerofoil section data, the treatment of the theory of aerofoil sections, extending over four chapters, makes it the latest comprehensive textbook, though mostly referring to German authors. The solution to the problem of finding the velocity over an arbitrary shape (Zweite Hauptaufgabe der Profiltheorie), as developed by the author, is given, as well as the results of the inverse problem (Erst Hauptaufgabe der Profiltheorie), as they are required for practical application. Also included in these chapters is an extensive treatment of viscous flows and of the effects arising from compressibility. The chapter dealing with compressible flows is limited to basic theories.

Apart from the undeniable value of this book in providing an efficient tool for the design of aerofoil sections, the amount of information collected on theoretical and experimentally developed sections under various conditions of flow is impressive. At the same time, particularly in view of the effects of viscosity, the book is a reflection of the attraction that lies in predicting nothing but the simple shape of wing sections which would enable us to achieve in practice conditions often so well displayed in experiments with two-dimensional aerofoil sections or flat plates. However, with a word or two on the finite span effects over aerofoils of modern plan form, the book would have approached nearer to practical wing design, as effects of this nature must be expected to upset the most ambitious calculations on aerofoil characteristics in two-dimensional viscous and compressible flow. The same uncertainty applies to boundary layer control, which may be considered a more drastic method of achieving improved conditions. High efficiency and tolerable complexity of equipment depend, here again, on the knowledge of the viscous flows over the wing.

Studying the final chapter, which is entirely devoted to graphs, essentially pressure distributions and polar diagrams, some of the figures, which are without reference

except as to the wind tunnel, lose value and purpose, particularly as reference in the text is not readily found. In general, the handling of some of the information would be easier, especially for non-German readers, if the notation were listed and the list of abbreviations were more complete.

However, such objections disappear when placed in the perspective of the material presented in the book with great discipline of mind.

The roughness of the paper makes reading pleasant, but quickly collects dust and fingerprints.—E. S. KRAUSS.

THE PERFORMANCE OF METAL-CUTTING TOOLS. *R. Tourret. Butterworths Scientific Publications, London, 1958. 183 pp. Illustrated. 50s.*

The Foreword of this book states that an attempt has been made to gather together published information and to show how that information may be correlated.

This is a modest statement considering that no less than 158 references on the subject of tooling and allied works are referred to in the 173 pages of what is undoubtedly a valuable contribution concerning a subject which very often is not properly understood by the apprentice and journeyman and in many cases only to a slightly greater extent by the experienced engineer.

The reader will appreciate that the work is not a practical workshop manual—as the author states, although the book does provide for considerable practical application.

The illustrations and descriptive matter concerning the type of cutting operations in common use, tool nomenclature and chip formation referred to in the introductory chapter form the basic data for the remaining sections and must be carefully read in order that the remainder of the work can be usefully understood.

The author's method of presentation, however, simplifies the effort such that even those readers with an elementary knowledge of machine tool cutting operations will readily understand the technicalities involved.

The reviewer does, however, consider that a companion volume providing data to assist the designer and operator on the practical and economic aspects of machining based upon the references and advice provided would prove of inestimable value and hopes that the author's statement (page 1) is not to be taken to mean that the engineer will always have to look elsewhere for such a book.

Summarising, the book provides data in a condensed form, much of which is taken from the writings of many authors on the subject of machine operations. The result is to provide a fairly quick means of reference for those who wish to improve the product to which the machine tool is applied.—R. E. FLAXMAN.

LOW LEVEL MISSION. *Leon Wolff. Longmans Green, London, 1958. 240 pp. Illustrated. 18s.*

Leon Wolff is a very competent writer of drama. As a historian he is less distinguished. The book concerns the series of attacks by the United States Air Force on the Ploesti group of oil refineries in Rumania from August 1943 to August 1944. The title suggests only one attack and, indeed, the low level mission was the first and most disastrous of them all. Here is drama, but it is all over by page 180 and the remainder is dull history of the misfortunes of the German oil industry, suffered chiefly at the hands of the Americans.

Had the author stuck to his drama he would have written an exciting book. He describes air combats and other occupational hazards very well but, as the blazing wrecks of oil refineries and tail-less bombers smoulder into lifelessness he lapses into the rather unbalanced style of the propagandist, the inevitable anti-climax emphasising the errors in his summing-up.

This purports to be a history of the air war against German oil, but the facts, though clearly (and, no doubt accurately) quoted in great detail, leave such obvious omissions that his conclusions spoil a book which might have been a useful essay on the subject. To quote an example, "The British damaged vast areas, but even radar methods produced an abysmally low ratio of crucial hits. Though still inefficient . . . the Americans were far more accurate." Mr. Wolff draws no attention to the British Pathfinder technique of target marking and the attacks on target indicators by individual bombers in the Main Force, nor does he explain how the Americans, flying in huge formations, attacked small targets without destroying large areas. And again, "The evidence is clear that the massive British bombing of cities was ill-advised militarily, regardless of its moral aspects. The effects were usually brief and shallow." Compared with American pattern bombing the results in human suffering were horribly similar. As for the morals of air warfare—Hm.—A.S.C.L.

Additions to the Library

Airways Modernization Board, The—its Mission and Methods. Franklin Institute. 1958. 160 pp. 4 dollars. This is the proceedings of a Symposium sponsored by the Franklin Institute in December 1957 giving 14 papers on air traffic control. The A.M.B. was set up in the Spring of 1957 and the Institute thought a symposium on its objectives would enable the newly-constituted Board to explain its functions and enable the public to express criticisms and views.

Basic Aeronautical Science and Principles of Flight. R. D. Blacker. Technical Press, London. 1958. 242 pp. Illustrated. 50s. *To be reviewed.*

British Naval Aircraft 1912-58. O. Thetford. Putnam. 1958. 426 pp. Illustrated. 50s. *To be reviewed.*

C.F.S. Birthplace of Air Power. J. W. R. Taylor. Putnam. 1958. 222 pp. Illustrated. 21s. *To be reviewed.*

Festschrift. Jakob Ackeret. Zeitschrift fur angewandte Mathematik und Physik. Birkauser Verlag, Basel. 1958. 777 pp. Diagrams. Swiss FR.24. *To be reviewed.*

Heat Exchangers—Applications to Gas Turbines. W. Hryniszak. London. 1958. 338 pp. Diagrams. 63s. *To be reviewed.*

Mass Analysis of Flames and Flue Gases. E. A. Bunt. University of Witwatersrand. 1958. 31 pp. A department of mechanical engineering report extending previous work to demonstrate the potentialities of direct sampling of gases withdrawn from constant pressure flames and exhaust ducts using a Nier-type spectrometer.

Mathematics of Physics and Modern Engineering. I. S. Sokolnikoff and R. M. Redheffer. McGraw-Hill, London. 1958. 810 pp. Diagrams. 74s. A successor to Sokolnikoff's *Higher Mathematics for Engineers and Physicists*, this book aims "To provide a sound and inspiring introduction to applied mathematics." Subjects included are: use of complex forms of solutions for linear differential equations, mean and pointwise convergence of Fourier series, topics concerned with the Poisson and Helmholtz equations, a modern treatment of probability and relative frequency, effective uses of complex variables in boundary-value problems, a meaningful discussion of Dirac's function in connection with the Laplace transform, an accessible treatment of matrices, vector spaces and field theory, relation

between differential and difference equations, and other topics in numerical analysis. Problems and worked examples are included.

Missile Engineering Handbook. C. W. Besserer. Van Nostrand, London. 1958. 600 pp. Illustrated. 109s. *To be reviewed.*

Proceedings of the 8th Symposium in Applied Mathematics of the American Mathematical Society. Vol. VIII. Calculus of variations and its applications. American Mathematical Society. McGraw-Hill, New York. 1958. 151 pp. 58s. This symposium, held in 1956 at the University of Chicago, was devoted to a consideration of the calculus of variations and its applications. The applications discussed were related to elasticity and plasticity, hydrodynamics, and dynamic programming. The contributors include Eric Reissner, D. C. Drucker, Joseph B. Keller, J. B. Diaz, J. L. Synge, M. M. Schiffer, Richard Bellman, S. Chandraesekhar and E. H. Rothe.

Rockets and Missiles. M. Allward and J. W. R. Taylor. Ian Allan. 1958. 36 pp. Illustrated. 2s. 6d. This is another of the Allan A. B. C. Series that appear with amazing frequency to the delight of spotters young and old. This one deviates from the usual pattern in that Mr. Taylor has not done a solo effort and also the subject is not likely to provide much opportunity to spotters. An introduction gives a startling comparison with the previous edition of only $2\frac{1}{2}$ years ago. In that time top speed has increased over threefold and altitude sixteen times. Rocket types have grown from 28 to nearly 100. A list at the end of missiles under development gives an amusing insight into the imaginative nomenclature of the U.S.A.

Three Hundred Years of Vibration Engineering. S. J. Zand. Birkhauser Verlag, Basel. 11 pp. This is a reprint from a commemorative number of Zeitschrift fur angewandte Mathematik und Physik prepared to celebrate the 60th birthday of Jakob Ackeret.

Turbo-Prop versus Turbo-Jet. F. H. Robertson. 1958. 11 pp. 12 Figures. A paper presented at the Society's Third Air Transport Course. Copies may be obtained from the author, Short Brothers and Harland Ltd., Queens Island, Belfast.

Reports

AERODYNAMICS

Boundary Layer—*see also* Control Surfaces

Effects of fabrication-type roughness on turbulent skin friction at supersonic speeds. K. R. Czarnecki et al. N.A.C.A. T.N. 4299. *July* 1958.—(1.1.1.4).

Effect of favorable pressure gradients on transition for several bodies of revolution at Mach 3·12. *J. R. Jack. N.A.C.A. T.N.* 4313. *July* 1958.
Transition locations on two constant-pressure-gradient models and two power-profile models were compared with those from a conventional cone.—(1.1.2.4).

Compressible Flow

A body modification to reduce drag due to wedge angle of wing with unswept trailing edge. W. C. Pitts and J. N. Nielsen. N.A.C.A. T.N. 4277. *July* 1958.
A term in Ward's slender body theory drag formula that is neglected in engineering application of the transonic area rule is considered. This term depends only upon the cross-sectional shape and surface slopes at the model base, and is therefore independent of the cross-sectional area distribution.—(1.2.2.1).

*Notes on the use of artificial distributions of singularities in supersonic minimum drag problems. E. W. Graham. Douglas Report SM-*23022. *December* 1957.
There are now several methods available for reducing certain supersonic minimum drag problems in three dimensional space to two dimensional potential flow problems. One of these methods involves the use of artificial distributions of singularities, which are so arranged as to produce the optimum combined flow field . Some of the characteristics of these artificial distributions are discussed and their use illustrated.—(1.2.3.1).

Discussion following the presentation of papers on hypersonics. AGARD Reports 132-147. *July* 1957.—(1.2.3).

Experimentelle Untersuchungen an verschieden stark konvergenten, schlanken Rotationakörpern bei mässig hohen Überschallgeschwindigkeiten. H. R. Voellmy. Mitt. Inst. fur Aero. Nr. 24. (*In German*).—(1.2.3).

Control Surfaces

Pressure-plotting measurements on an 8 *per cent thick aerofoil with trailing-edge flap blowing. J. Williams and A. J. Alexander. R. & M.* 3087. 1958.
Wind tunnel experiments were made on an 8 per cent thick aerofoil between end plates, with blowing from a slot in the knee of a 25 per cent chord trailing-edge flap, to improve the lifting efficiency of the flap. Both the blowing-slot width and position were varied. The sectional lift and pitching moment were derived by chordwise integration of the surface static pressures measured at the mid-span station. Tuft observations as well as surface-pressure measurements were made to determine the extent of the turbulent separation region on the trailing-edge flap and of the liminar separation bubble on the aerofoil nose.—(1.3.4×1.10.2.1).

Some contributions to jet-flap theory and to the theory of source-flow from aerofoils. L. C. Woods. C.P. 388. 1958.
A theoretical study is presented of the thrust, lift and moment on an aerofoil due to a two dimensional jet of air ejected from the trailing edge at an angle to the main stream.—(1.3.4× 1.10.1.1).

Full-scale wind tunnel tests of a 35° *sweptback wing airplane with blowing from the shroud ahead of the trailing-edge flaps. W. H. Tolhurst. N.A.C.A. T.N.* 4283. *July* 1958.
Data are presented showing the effect of flap position relative to the blowing nozzle on the air flow requirements for boundary layer control on flaps deflected from 45° to 75° at a Reynolds number of $7{\cdot}5\times10^6$. A direct comparison of shroud blowing and flap blowing is shown for a plain flap configuration. Also presented are data showing the effect on the air flow requirements of spacers in the nozzle, discontinuities on the flap upper surface, and of sealing the slot when the flap was in the single-slotted flap configuration.—(1.3.4×1.1.6.1).

Internal Flow

Flow in a jet directed normal to the wind. R. Jordinson. R. & M. 3074. 1958.
Experiments to study the flow in an air jet, which is ejected normal to an air stream from an orifice in a plane floor, have been made for several values of the ratio of wind velocity to jet exit velocity. The path of the jet and the shape of the jet cross-section were determined.—(1.5.1).

Design and tests of a six-stage axial-flow compressor having a tip speed of 550 *feet per second and a flat operating characteristic at constant speed. W. R. Westphal and J. W. Maynard. N.A.C.A. T.N.* 4253. *June* 1958.
The six-stage axial-flow compressor was designed for a constant power input per pound of flow. Several configurations having different blade setting angles and solidities were tested.—(1.5.2×27.1).

Stability and Control

Dynamic stability of vehicles traversing ascending or descending paths through the atmosphere. M. Tobak and H. J. Allen. N.A.C.A. T.N. 4275. *July* 1958.
An analysis is given of the oscillatory motions of vehicles which traverse ascending and descending paths through the atmosphere at high speed. The specific case of a skip path is examined in detail, and this leads to a form of solution for

the oscillatory motion which should recur over any trajectory.—(1.8.2.1×25.2).

Flight investigation of the acceptability of a small side-located controller used with an irreversible hydraulic control system. H. A. Kuehnel and R. W. Sommer. N.A.C.A. T.N. 4297. *July* 1958.
Results indicate that the side location of the controller is consistent with pilot comfort and the aeroplane is flyable with this type of controller. Results are based on the opinions of six pilots and are supported by time-history records of typical flight runs.—(1.8×9).

Wind tunnel investigation of the high subsonic static longitudinal stability characteristics of several wing-body configurations designed for high lift-drag ratios at a Mach number of 1·4. *P. G. Fournier. N.A.C.A. T.N.* 4340. *July* 1958.
The Mach number range was 0·60 to 0·95 with a corresponding Reynolds number of $2{\cdot}0\times10^6$ to $3{\cdot}3\times10^6$ based on mean aerodynamic chord. The basic highly swept (approximately 60° at $c/4$) wing had a taper ratio of 0·167, an aspect ratio of 2·91, and thin N.A.C.A. 65A series aerofoil sections. Other wings, of the same plan form, had twist and camber and twist.—(1.8.2.2).

A search for definitions and criteria relating to non-oscillatory divergence in near steady flight conditions. E. Billion. AGARD Report 72*T*. *August* 1958.
The method generally applied is exploited for the various basic conditions likely to be encountered: straight flight (at any inclination) in the earth's field of gravity (longitudinal and transverse behaviour); curvilinear flight outside the earth's field of gravity (applied to the problem of pitch-up); the most general steady flight condition (helix traced on a cylinder of revolution with a vertical axis) in the earth's field of gravity.—(1.8).

THERMO-AERODYNAMICS

Heat transfer in isotropic turbulence during the final period of decay. D. W. Dunn and W. H. Reid. N.A.C.A. T.N. 4186. *June* 1958.—(1.9.1).

Effects of boundary layer displacement and leading-edge bluntness on pressure distribution, skin friction, and heat transfer of bodies at hypersonic speeds. M. H. Bertram and A. Henderson. N.A.C.A. T.N. 4301. *July* 1958.
Results are presented of an investigation to determine the effect of boundary layer displacement and leading-edge bluntness on surfaces in hypersonic flow. Methods for predicting these phenomena on two-dimensional surfaces are given with a brief review of recent three-dimensional results.—(1.9.1×1.10).

Prandtl number effects on unsteady forced-convection heat transfer. E. M. Sparrow and J. L. Gregg. N.A.C.A. T.N. 4311. *June* 1958.
An analysis is made for laminar forced-convection heat transfer on a flat plate with unsteady surface temperature. The deviation of the instantaneous heat transfer rate from the quasi-steady value is computed. Results are obtained for Prandtl numbers in the range 0·01 to 10.—(1.9.1).

WINGS AND AEROFOILS—*see also* CONTROL SURFACES
THERMO-AERODYNAMICS

A theoretical and experimental investigation of the flow over a family of rectangular wings of biconvex section at $M=1{\cdot}42$. *R. C. Lock. R. & M.* 3055. 1958.
An account is given of experiments made at $M=1{\cdot}42$ using three rectangular half-wing models having biconvex sections with thickness ratios 0·04, 0·06 and 0·08, mounted on a reflection plate. Measurements were made of the pressure on the upper surface of the wing and of pressure and flow direction in the neighbourhood of the wing tip. Direct shadow photography and observation of surface oil patterns enabled various details of the flow to be visualised.—(1.10.1.2×1.10.2.2).

Experiments on the effect of air jets and surface roughness on the stalling characteristics of a 5 ft. chord N.A.C.A. 64A006 aerofoil. D. G. Hurley et al. ARL Report A. 107. *December* 1957.—(1.10.2.1).

HELICOPTER AERODYNAMICS

Flow induced by a rotor in power-on vertical descent. W. Castles. N.A.C.A. T.N. 4330. *July* 1958.
Approximate equations are derived for the induced power required and blade loading of a lifting rotor operating in the power-on vertical descent range. The approximate relations, which are based upon certain assumptions as to the nature of the flow pattern, yield, for the induced power variation, results which are in general agreement with available data.—(1.11.3).

TESTING AND INSTRUMENTS

Boundary-induced downwash due to lift in a two dimensional slotted wind tunnel. S. Katzoff and R. L. Barger. N.A.C.A. T.N. 4289. *June* 1958.
A solution has been obtained for the complete tunnel interference flow for a lifting vortex in a two dimensional slotted tunnel. Curves are presented for the longitudinal distribution of tunnel induced downwash angle for various values of the boundary openness parameter and for various heights of the vortex above the tunnel centre line.—(1.12.1).

AEROELASTICITY

An evaluation of effects of flexibility on wing strains in rough air for a large swept-wing airplane by means of experimentally determined frequency-response functions with an assessment of random-process techniques employed. T. L. Coleman et al. N.A.C.A. T.N. 4291. *July* 1958.
Flight test measurements on a large aeroplane are analysed to determine the effects of aeroplane flexibility on wing bending and shear strains. The power spectra of the strain responses and the frequency-response functions for the strain responses to vertical gust disturbances are determined and compared with the strain responses for a quasi-rigid aeroplane. A general analysis of the reliability of the results is presented and methods of estimating the distortions and sampling errors are developed. These methods are applied to the interpretation of the test results.—(2×33.2.3.2).

AIRPORTS

See STRUCTURES—LOADS

AVIATION MEDICINE

See also AERODYNAMICS—STABILITY AND CONTROL

Measurement of pilot mental effort. S. I. Cohen and A. J. Silverman. AGARD Report 148. *May* 1957.
The factors are discussed which affect a pilot's ability to respond to situations demanding effort and methods of measuring his total psychophysiologic response. A description is given of some methods and quantitative results obtained in research.—(9).

HYDRODYNAMICS

High-speed hydrodynamic characteristics of a flat plate and 20° *dead-rise surface in unsymmetrical planing conditions. D. Savitsky et al. N.A.C.A. T.N.* 4187. *June* 1958.
The results of an investigation made to obtain the wetted areas, the three components of planing forces, and the three components of moments acting on a 0° and a 20° dead-rise surface in high-speed, unsymmetrical planing conditions.—(17.2).

Effects of nose shape and spray control strips on emergence and planing spray of hydro-ski models. J. R. McGehee. N.A.C.A. T.N. 4294. *July* 1958.
The emergence-spray characteristics of the models with various bow shapes were investigated for depths of submersion of the bow from 0·50 inch to −0·50 inch. The planing-spray characteristics were determined for a draft corresponding to a length-beam ratio of 4. This investigation was made for trims of 12° and 20° at a speed of 30 feet per second. The emergence-spray and planing-spray patterns are shown in photographs.—(17.2).

MATERIALS

Compressive strength and creep of 17-7*PH stainless-steel plates at elevated temperatures. B. S. Stein. N.A.C.A. T.N.* 4296. *July* 1958.
Compressive strength test results from room temperature to 1,000°F and compressive creep test results from 700°F to 1,000°F are presented for plates of 17-7PH stainless steel, condition TH 1,050, which were edge-supported in V-groove fixtures. Plate width-thickness ratios range from 15 to 60.—(21.2.1).

The upper temperature limit of stability of Guinier-Preston zones in ternary aluminium-zinc-magnesium alloys. I. J. Polmear. ARL Note MET. 9. *March* 1958.
The metastable boundary for Guinier-Preston (G.P.) zone formation in ternary aluminium-zinc-magnesium alloys has been determined and it appears as a continuous surface on the phase diagram. This suggests that G.P. zone formation is rather similar over a wide range of composition. The results also suggest that, for alloys with a high ratio of magnesium to zinc, the processes of the age hardening and solid solution hardening may both be significant.—(21.2.2).

Transparent materials for aircraft—a review. E. W. Russell. AGARD Report 160. *November* 1957.
Developments in transparent materials for aircraft are reviewed. Extensive work has been done on the organic plastics, and offers improved performance and reliability, but with only limited advances in high temperature performance. As alternative materials the inorganic glasses compel attention but suffer from several limitations both for windscreens and canopies. Possible lines of development are indicated briefly.—(21.3).

Cinétique du développement d'un gros cristal dans la méthode de l'allongement critique. H. Latiere. Pubs. Sc. et Tech. 342. 1958. (*In French*).—(21.1).

The effect of the relative humidity of the air during fabrication on the mechanical properties of glass fabric reinforced plastics. A. Hartman. NLL. T.N. M. 2045. *January* 1958.
Using a glass fabric and polyester-resin, the effect of temperature and relative humidity during curing were investigated. Results are presented for porosity, structure, mechanical properties and reduction in mechanical properties due to water immersion.—(21.3.3).

MATHEMATICS

See also—MECHANICAL ENGINEERING

Operational formulae for response calculations. S. Neumark. R. & M. 3075. 1958.
Systematic tables are presented of formulae whose purpose is to facilitate the operational solution of response problems reducible to linear differential equations with constant coefficients and with simple forcing functions. The formulae enable the user to find operational equivalents of a wide class of simple functions and, inversely, to find functional equivalents of a great number of operational expressions, in the most rapid and direct manner. The introductory text describes the method of deriving the formulae and explains how to use them in solving response problems. A number of examples show the advantages of the tables and give solutions of several typical problems.—(22.1).

MECHANICAL ENGINEERING

Spectral densities dependent on two frequencies and their use in the study of linear, stochastic differential equations with time-dependent coefficients. M. Sundström and K. Flodin. K.T.H. Aero T.N. 43. 1957.
A power spectral density depending on two frequencies is defined for use in the study of servo systems described by linear differential equations with time-dependent coefficients. For stationary processes, these spectral densities are computed from the ordinary auto-correlation and cross-correlation functions. A simple numerical example is treated.—(23.3×22).

MISSILES

See AERODYNAMICS—STABILITY AND CONTROL

POWER PLANTS

See also AERODYNAMICS—INTERNAL FLOW

Near noise field of a jet engine exhaust. W. L. Howes et al. N.A.C.A. Report 1338. (*Supersedes N.A.C.A. T.N.* 3763 *and* 3764.) 1957.
Surveys of sound pressures and longitudinal and lateral cross correlations, both overall and in frequency bands, are presented for a fixed operating condition. The dependence on jet velocity and the effect on a stiff plate are shown. Sound pressure data with after-burning are included. The distribution of acoustic sources according to frequency was found. The physical significance of cross correlation is outlined.—(27.1×32.2.3).

PROPELLERS

Flight measurements of the vibratory bending and torsional stresses on a modified supersonic propeller for forward Mach numbers up to 0·95. *T. C. O'Bryan. N.A.C.A. T.N.* 4342. *June* 1958.
Vibratory bending stresses, measured in flight, on a modified supersonic propeller were low in relation to the strength of the material. The vibratory torsional stresses did not exceed ±1,000 lb./in^2.—(29.7).

FATIGUE

An hydraulic random load fatigue test rig for simple specimens. A. D. McEwan et al. ARL Note SM. 245. *April* 1958.
A rig is described in which "chains" of specimens in axial loading may be subjected to a spectrum derived from gust loading data for civil transport aircraft. This is used in conjunction with a hydraulic rig suitable for applying a random load spectrum to complete aircraft wings.—(31.2.1.1.3).

The effect on the static and fatigue properties of riveted light alloy lap joints of reinforcing the critical section with thin adhesive bonded sheets. A. Hartman and P. de Rijk. NLL T.N. M. 2047. *March* 1958.
Static and fatigue tests in fluctuating tension have been executed on two row riveted lap joints in aluminium alloy, reinforced over the rivets by adhesive-bonded strips of half the sheet thickness. Increases in static and fatigue strength are shown and compared with the calculated bending moments at the critical sections.—(31.2.3.2.2.2×33.2.4.13).

SCIENCE—GENERAL

See POWER PLANTS

STRUCTURES

LOADS

Measurements and power spectra of runway roughness at airports in countries of the North Atlantic Treaty Organisation. W. E. Thompson. N.A.C.A. T.N. 4303. *July* 1958.
Measurements of runway roughness obtained by a profile-survey method are presented. A variety of runway lengths or sections and degrees of roughness were measured. The results are presented as elevation profiles of the runways surveyed and in the form of power spectra.—(33.1.2×6).

THEORY AND ANALYSIS—*See also* AEROELASTICITY
FATIGUE

Local instability of the elements of a truss-core sandwich plate. M. S. Anderson. N.A.C.A. T.N. 4292. *July* 1958.
The charts presented give the compressive buckling coefficients for a single-truss-core and a double-truss-core sandwich plate. These charts cover a wide range of sandwich proportions and may be used for sandwiches with unequal faces. They apply to inplane compressive loads acting parallel or perpendicular to the core direction or for various combinations of these loads. —(33.2.4.6.6).

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Geometric stress concentration factors. Notches in flat bars or strips
Geometric stress concentration factors. Diametral holes in solid circular rods
Geometric stress concentration factors. Circumferential grooves in solid circular rods
Geometric stress concentration factors. Fillets in a flat bar or strip in tension
Geometric stress concentration factors. Loaded pin in a central circular hole in a flat bar
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1. Guided Weapons and Aircraft—Some Differences in Design and Development, by J. E. Serby, C.B., C.B.E., B.A., F.R.Ae.S., Director-General of Guided Weapons, Ministry of Supply. March 1958.
2. Guidance and Control, by L. H. Bedford, C.B.E., M.A., B.Sc., Chief Engineer, Guided Weapons Division, English Electric Co. Ltd. May 1958.
3. Guided Flight Trials, by R. W. M. Boswell, O.B.E., M.Sc., Deputy Controller (Trials and Instrumentation) Weapons Research Establishment, South Australia. June 1958.
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